ATMOSPHERIC NEUTRINOS

Large uncertainties ~20% on absolute normalization but useful well known quantities can be singled out.
Atmospheric $\nu$ events

$$\nu_{\mu(e)} + N \rightarrow \mu^\pm (e^\pm) + X$$

**Super-Kamiokande response curves**

- $\nu_{\mu}$ and $\nu_e$
- Sub-GeV: $E_{\text{vis}} < 1.33$ GeV
  - $88\%$ CC $\nu_e$
  - $96\%$ CC $\nu_\mu$
- Multi-GeV: $E_{\text{vis}} > 1.33$ GeV
  - $84\%$ CC $\nu_e$
  - $99\%$ CC $\nu_\mu$
- $97\%$ $\nu_\mu$ CC $<E_\nu> \sim 10$ GeV
- $<E_\nu> \sim 100$ GeV
- $<E_\nu> \sim 10$ GeV
Parameters useful for $\nu$ oscillation studies

- Flavor ratio $\nu_e + 1/3 \bar{\nu}_e / \nu_\mu + 1/3 \bar{\nu}_\mu$ For $E_\nu < 30 \text{ GeV} \sim 5%$
- Up/down asymmetry
- Full angular distribution

From full simulations:

Earth spherical symmetry + CR flux isotropy
\[ \Phi(E_\nu, \theta) = \Phi(E_\nu, \pi - \theta) \]
Shape of the angular distribution of HE neutrinos

Uncertainties:
1) $\delta(V/H)/(V/H) \sim 0.12 \delta(K/\pi)/(K/\pi)$
$L_{\text{dec}} \sim 0.75 (E(\text{GeV})/100) \text{ km (K)}$
$L_{\text{dec}} \sim 5.6 (E(\text{GeV})/100) \text{ km (\pi)}$
almost all K decay up to high energies
($> 100 \text{ GeV}$) almost isotropic
competition of interaction/decay for $\pi^\pm$: decay more easily at horizon for increasing energy $\Rightarrow$ horizontal $>$ vertical flux.
Flux from Kaons isotropic up to energies higher than pions
2) $\delta(V/H)/(V/H) \sim 0.25 \delta \alpha$
uncertainty in the slope of primary flux
3) Seasonal variations
In quadrature: $\sim 3\%$ error on $V/H$

Useful for channel determination

\begin{align*}
\nu_{\mu} & \rightarrow \nu_{\tau} \\
\nu_{\mu} & \rightarrow \nu_{\text{sterile}}
\end{align*}
The atmospheric $\nu$ problem: measured flavor ratio

Flavor ratio:

$$R = \frac{\left(\frac{\mu - \text{like}}{e - \text{like}}\right)_{\text{DATA}}}{\left(\frac{\mu - \text{like}}{e - \text{like}}\right)_{\text{MC}}}$$

$\mu$-like (tracks): deficit
$e$-like (showers): in agreement with expected

Kamiokande Multi-GeV: flavor ratio angular dependence as expected from oscillations
Oscillations in Atmospheric Neutrinos

\[ 100 \text{ MeV} \lesssim E_\nu \lesssim 10 \text{ TeV} \]

\[ 10 \text{ km} \lesssim L \lesssim 10^4 \text{ km} \]

For Sub-GeV and Multi-GeV

\[ P(\nu_\ell \rightarrow \nu_\ell) = 1 - \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4E_\nu} \right) \]

\[ \langle P(L \leq 100 \text{ km}) \rangle \rightarrow 1 \]

\[ \langle P(L \geq 2000 \text{ km}) \rangle \rightarrow 1 - \frac{\sin^2 2\theta}{2} \Rightarrow \frac{1}{2} \]
SK results

\[
\frac{\langle \mu / e \rangle_{data}}{\langle \mu / e \rangle_{MC}} = 0.649 \pm 0.016 \pm 0.051
\]

\[
\frac{\langle \mu / e \rangle_{data}}{\langle \mu / e \rangle_{MC}} = 0.699 \pm 0.032 \pm 0.08
\]

Observed \( A_{\mu\text{-like}} \) 9.5\( \sigma \) from no-oscillation prediction!
MACRO at Laboratori Nazionali del Gran Sasso

77 x 12.3 x 9 m³
Run time: 1989-2000
Completed in 1994
Average rock coverage
3700 mwe

11 km-long highway tunnels
Tracking with Streamer Tubes

Streamer tube chambers:

- 20000 m² of 3x3 cm² x 12 m cells with 100µm Cu-Be wire
- Gas mixture: He + n-pentane (27%)
- Pick-up strips for stereo track reconstruction
- Intrinsic angular resolution ~0.2°
Neutrino events

\[ \nu \mu + p \rightarrow n + \mu^+ \]

Throughgoing
Internal Down
Internal Up

Detector mass \( \sim 5.3 \text{ kton} \)
Time Of Flight technique

Scintillators:

- 600 tons of liquid scintillator (mineral oil + pseudocumene + wls) in 12 m-long boxes;
- time resolution ~700 ps;
- calibration tools: atmospheric µs, Light Emitting Diodes, laser light;
- 200 MHz Wave Form Digitizers for pulse shape analysis;
Through going muons

Similar results for SK
The oscillation pattern

The binning choice is critical

SK $\nu_\mu$ compared to predictions for oscillations $\nu$-decay and decoherence

Hep-ex/0404034
K2K

KEK to Kamioka (L = 250 km): ν beam from 12 GeV protons accelerated by the KEK proton synchrotron on aluminium target
98% pure muon neutrinos with mean energy 1.3 GeV

Events in SK in time coincidence inside 1.5 μs (reduce atm ν background in 22.5 kton SK fiducial volume to 10^{-3})
Measured: 56 (Expected: 80.1^{+6.2}_{-5.4}) and in Feb 2004 108 measured 150.9 ± 11 predicted

Observables to infer oscillations: energy spectrum and normalization
Atmospheric neutrino results

![Graph showing atmospheric neutrino results.](image)
Results for atmospheric neutrinos

Figure 7: Left: 90% C.L. allowed region contours for $\nu_\mu \rightarrow \nu_\tau$ oscillations obtained by the Super-Kamiokande, MACRO and Soudan-2 experiments [29]. Right: Allowed region contours for $\nu_\mu$ disappearance obtained in the K2K experiment confronted with the allowed regions for $\nu_\mu \rightarrow \nu_\tau$ oscillations obtained in the Super-Kamiokande experiment [151].
Astrophysical Neutrino Oscillations

\[
U = \begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix} = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\
s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23}
\end{pmatrix}
\]

If \( \theta_{13} = 0 \) \( \Rightarrow \) \( c_{13} = 1 \) and \( s_{13} = 0 \) and \( \delta = 0 \) and for normal hierarchy

\[
U = \begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12}c_{23} & c_{12}c_{23} & s_{23} \\
s_{12}s_{23} & -c_{12}s_{23} & c_{23}
\end{pmatrix} = \begin{pmatrix}
c_{\text{sol}} & s_{\text{sol}} & 0 \\
-s_{\text{sol}}c_{\text{atm}} & c_{\text{sol}}c_{\text{atm}} & s_{\text{atm}} \\
s_{\text{sol}}s_{\text{atm}} & -c_{\text{sol}}s_{\text{atm}} & c_{\text{atm}}
\end{pmatrix}
\]

\( \theta_{12} \approx 35\,\text{deg} \Rightarrow c_{\text{sol}} = 0.82 \) and \( s_{\text{sol}} = 0.57 \)

\( \theta_{23} \approx 45\,\text{deg} \Rightarrow s_{\text{atm}} = c_{\text{atm}} = 1 \)

\[
U = \begin{pmatrix}
c & s & 0 \\
-sx & cx & x \\
sx & -cx & x
\end{pmatrix} = \begin{pmatrix}
0.82 & 0.57 & 0 \\
-0.4 & 0.58 & 1/\sqrt{2} \\
0.4 & -0.58 & 1/\sqrt{2}
\end{pmatrix}
\]
Astrophysical Neutrino Oscillations

\[ P(\nu_{\alpha} \rightarrow \nu_{\beta}) = \sum_{i,j} U_{\alpha,i} U_{\beta,i}^{*} U_{\alpha,j} U_{\beta,j}^{*} e^{-i \Delta m_{ij}^2 L / 2E} \]

\[ \Delta m_{sol}^2 \approx 8 \cdot 10^{-5} \text{eV}^2 \]
\[ \Delta m_{atm}^2 \approx 2.5 \cdot 10^{-3} \text{eV}^2 \]

If CP is conserved (\( \delta = 0 \)) this expression can be written as (U is a real matrix):

\[ P(\nu_{\alpha} \rightarrow \nu_{\beta}) = \sum_{i} |U_{\alpha,i}|^2 |U_{\beta,i}|^2 + 2 \sum_{i<j} U_{\alpha,i} U_{\beta,i}^{*} U_{\alpha,j} U_{\beta,j}^{*} \cos \left( \frac{\Delta m_{ij}^2 L}{2E} \right) \]

For astrophysical sources \( L > \text{kpc} \) and \( \Delta m^2 L / 2E \gg 1 \).

Let’s give a typical number to the phase factor for a source at a distance of 1 kpc emitting neutrinos of 10 TeV:

\[ \varphi = \frac{1.27 L(\text{km}) \Delta m_{ij}^2(\text{eV}^2)}{E(\text{GeV})} \approx \frac{1.27 \cdot 3.1 \cdot 10^16 \cdot 8 \cdot 10^{-5}}{10^4} \approx 3 \cdot 10^8 \]

\[ \varphi \sim 3 \cdot 10^8 \left( \frac{\Delta m^2}{8 \cdot 10^{-5} \text{eV}^2} \right) \left( \frac{D}{1 \text{kpc}} \right) \left( \frac{10 \text{TeV}}{E_{\nu}} \right) \]

Let us assume that an experiment measures the events in a small energy bin so that we can consider approximately constant the energy \( E \), then the oscillating term is given by \( \text{const} \times \cos L \), so the term averages to zero. As a matter of fact, his value means that if the distance of the source (or eventually the energy) of the emitted neutrinos is not known with a precision of \( 10^8 \) the oscillating term averages to zero. Since sources have extensions of about 1 pc and their distance is > 1 kpc their distance are known with precision 1/1000!! Also the energy is about 30% uncertain.
Astrophysical Neutrino Oscillations

Hence for astrophysical sources $L>kpc$: the uncertainties on distances to sources and on their dimensions eliminate the effect of the phase term.

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sum_i |U_{\alpha,i}|^2 |U_{\beta,i}|^2$$

Eg.

$$P(\nu_e \rightarrow \nu_e) = \sum_i |U_{e1}|^2 |U_{e1}|^2 = |U_{e1}|^4 + |U_{e2}|^4 + |U_{e3}|^4 = 0.82^4 + 0.57^4 + 0 = 0.56$$

$$P(\nu_e \rightarrow \nu_\mu) = \sum_i |U_{e1}|^2 |U_{\mu1}|^2 = |U_{e1}|^4 |U_{\mu1}|^4 + |U_{e2}|^4 |U_{\mu2}|^4 + |U_{e3}|^4 |U_{\mu3}|^4 = 0.82^2 \cdot 0.4^2 + 0.57^2 \cdot 0.58^2 + 0 = 0.22$$

$$P(\nu_e \rightarrow \nu_\tau) = \sum_i |U_{e1}|^2 |U_{\tau1}|^2 = |U_{e1}|^4 |U_{\tau1}|^4 + |U_{e2}|^4 |U_{\tau2}|^4 + |U_{e3}|^4 |U_{\tau3}|^4 = 0.82^2 \cdot 0.4^2 + 0.57^2 \cdot 0.58^2 + 0 = 0.22$$

<table>
<thead>
<tr>
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Astrophysical Neutrino Oscillations

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So for $\nu_e: \nu_\mu: \nu_\tau = 1:2:0$ : for $\nu_e$ 60% comes from $\nu_e$ survival and $2 \times 20\%$ from $2 \nu_\mu$ conversion =>100%. For $2 \nu_\mu 2 \times 40\% = 80\%$ comes from $\nu_\mu$ survival, then 20% from $\nu_e$ that become $\nu_\mu$ => 100%

$\nu_\tau$ will appear after 20% of $\nu_e + 2 \times 40\%$ of $\nu_\mu = 100\%$

For $n$ decay $\quad n \rightarrow p + e^- + \bar{\nu}_e$ from the Galactic Centre at L~10 kpc anti-electron neutrinos convert according the same matrix into 20% muon neutrinos and 20% tau neutrinos. And 60% electron neutrinos will remain such.
Suggested Readings

• Textbooks
Halzen and Martin, Quarks and Leptons, An Introductory Course to Modern Physics, Wiley 1984
B.R. Martin and G. Shaw, Particle Physics, Manchester Physics Series (1987)
Perkins, Introduction to High Energy Physics, Addison-Wesley, 1987
L. Bergstrom and A. Goobar, Cosmology and Particle Astrophysics (2nd edition), Springer 2004 cap 6

Neutrino people do not miss
http://www.nu.to.infn.it/
http://www.nu.to.infn.it/pap/0310238/ (neutrino mixing)
http://prola.aps.org/abstract/PRD/v57/i7/p3873_1
http://pdg.lbl.gov/2005/reviews/solarnu_s005313.pdf (solar neutrinos)