Physics 202 Midterm Exam 2
October 29, 2007

Name: Key | ID#:

Section:

TA (please circle):

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Instructions:

1. Don't forget to write down your name and section number.
2. **Show your work! A reasonable amount of work is required to receive full credit.**
3. Be aware that intermediate steps earn points even if the final answer is incorrect.
4. Erase (or cross out) any mistakes or you will be marked down. Grading is based on everything you have written down.
5. Both the magnitude and direction of vector quantities need to be specified for full credit.

Fundamental Constants:

$\varepsilon_0 = (4\pi k_e)^{-1} = 8.85\times10^{-12}\,\mathrm{C^2/(N\cdot m^2)}$ $\mu_0 = 4\pi\times10^{-7}\,\mathrm{T\cdot m/A}$

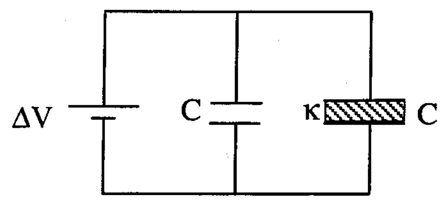
$m_p = 1.67\times10^{-27}\,\mathrm{kg}$ $m_e = 9.11\times10^{-31}\,\mathrm{kg}$ $q_p = -q_e = 1.6\times10^{-19}\,\mathrm{C}$

Scores:

Problem 1 ____________ **Problem 2** ____________ **Problem 3** ____________

Problem 4 ____________ **Problem 5** ____________

1. Two parallel-plate capacitors, each with capacitance C, are connected in parallel across a battery. Without disconnecting the battery, a slab with dielectric constant κ is inserted between the plates of one of the capacitors, completely filling the gap.



(a) What is the new effective capacitance of the combination of capacitors after the dielectric is inserted? (5 points)

$C_{eq} = C_1 + C_2$ $\quad C_1 = C \quad C_2 = kC$

$\boxed{C_{eq} = C(1+k)}$

(b) Determine the charge on each capacitor both before and after the dielectric is inserted. (10 points)

Before: $\boxed{Q_1 = Q_2 = C\Delta V}$

After: $\boxed{Q_1 = C\Delta V}$ $\boxed{Q_2 = kC\Delta V}$

(c) What is the change in the total energy stored in the combination of capacitors after the dielectric is inserted? (5 points)

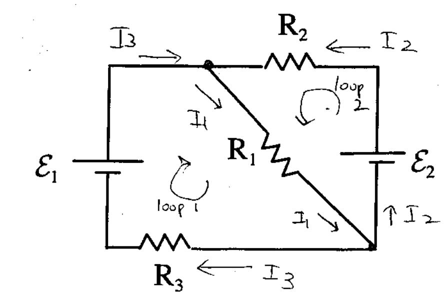
Initial $\rightarrow$ $U_i = 2\left(\frac{1}{2}C(\Delta V)^2\right)$

Final $\rightarrow$ $U_f = \frac{1}{2}C(\Delta V)^2 + \frac{1}{2}kC(\Delta V)^2 = \frac{1}{2}C(1+k)(\Delta V)^2$

$\Delta U = U_f - U_i = \left(\frac{1}{2}C(1+k) - C\right)(\Delta V)^2$

$\boxed{\Delta U = \frac{(k-1)}{2}C(\Delta V)^2}$

2. In the circuit given below, $\mathcal{E}_2 = 2\mathcal{E}_1 = 50$ V, and $R_1 = R_2 = R_3 = 100\,\Omega$.



(a) Determine the current flowing through each branch of the circuit. (10 points)

I assigned currents as above, such that I_1 is the current flowing through R_1, I_2 through R_2, I_3 through R_3

Junction rule: $I_1 = I_2 + I_3$

loop 1: $\mathcal{E}_1 - I_1 R - I_3 R = 0 \Rightarrow \mathcal{E}_1 - 2I_1 R + I_2 R = 0.$

loop 2: $\mathcal{E}_2 - I_2 R - I_1 R = 0$

Solve to find: $\boxed{I_1 = \frac{\mathcal{E}_1 + \mathcal{E}_2}{3R} = \frac{\mathcal{E}_1 + 2\mathcal{E}_1}{3R} = \mathcal{E}_1/R}$ $\boxed{I_2 = \frac{2\mathcal{E}_2 - \mathcal{E}_1}{3R} = \mathcal{E}_1/R}$ $\boxed{I_3 = I_1 - I_2 = 0}$

$\Rightarrow \boxed{I_1 = 25\text{ V}/100\,\Omega = .25\text{ A} = I_2 \quad I_3 = 0}$.

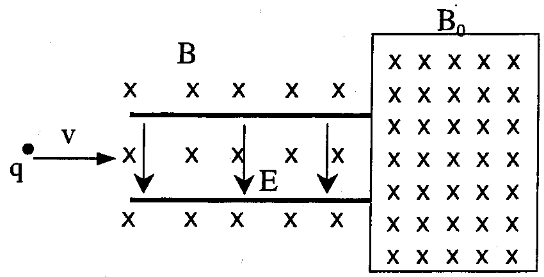
(b) Compute the power delivered to R_1. (5 points)

Power to R_1: $I_1^2 R = \left(\frac{\mathcal{E}_1}{R}\right)^2 R = \frac{\mathcal{E}_1^2}{R} = \frac{(25\text{ V})^2}{100\,\Omega} = \boxed{6.25\text{ W}}$

(c) If each resistor consists of a length of Constantan wire (resistivity $\rho = 4.9\times10^{-7}\,\Omega\cdot\text{m}$) of diameter $d = 1$ mm wrapped around an insulating core, what is the length of the wire? Note: this problem can be solved independently of parts (a) and (b). (5 points)

$R = \rho\ell/A \rightarrow \ell = \frac{AR}{\rho} = \frac{\pi(d/2)^2 R}{\rho} = \frac{\pi\left(\frac{1}{2}\times10^{-3}\text{ m}\right)^2 (100\,\Omega)}{4.9\times10^{-7}\,\Omega\text{ m}} = \underline{160.3\text{ m}}$

3. Before entering a mass spectrometer, positively charged ions pass through a velocity selector. The velocity selector consists of parallel plates separated by 3 mm with a potential difference of 180 V. The magnetic field strength in the region between the plates has the magnitude B = 0.4 T. The magnetic field strength in the mass spectrometer is B_0 = 1.2 T.



(a) Indicate the directions of the forces on the ion in the velocity selector. (5 points)

$q \; \xrightarrow{\vec{v}}$ $\quad \uparrow \vec{F}_B \quad \downarrow \vec{F}_E$ since $q > 0$.

(b) Find the speed of the ions entering the mass spectrometer. (5 points)

$qE = qvB \Rightarrow$

$$v = E/B = \frac{\left(\frac{180\,V}{3\times10^{-3}\,m}\right)}{0.4T} = \boxed{1.5\times10^{+5}\ m/s}$$

units → recall $[E] = N/C$

$[B] = N/(C\,m/s) \rightarrow [E]/[B] \rightarrow m/s$.

(c) What is the radius of the orbit of a singly ionized ^{235}U ion, which has a mass of 3.903×10^{-25} kg? (10 points)

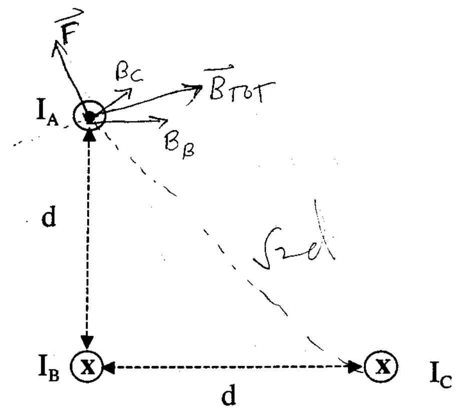
$$qvB_0 = \frac{mv^2}{r}$$

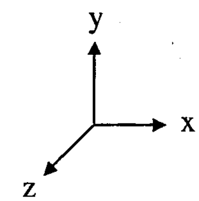
$$\Rightarrow \quad r = \frac{mv}{qB_0} = \frac{m(E/B)}{qB_0} = \frac{(3.903\times10^{-25}\,kg)\,(1.5\times10^5)\,m/s}{(1.6\times10^{-19}\,C)(1.2\,T)}$$

$$\boxed{r = .3\ m}$$

units →
$$\frac{kg\ m/s}{C\ T} \rightarrow \frac{kg\ m/s}{C\ \frac{N}{C\,m/s}} \rightarrow \frac{kg\ m^2/s^2}{N} \Rightarrow \frac{kg\ m^2/s^2}{kg\ m/s^2} \rightarrow m.$$

4. Three long, parallel straight wires pass through the three of the vertices of an square with sides d=12 cm. The current flowing through each wire is 10 A. The directions of the current flow are either into the page or out of the page (the z direction), as indicated on the figure.



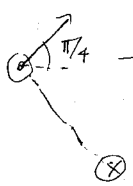


(a) Give the x and y components of the magnetic field due to I_B at the location of I_A. (5 points)

$$\vec{B}_B = \frac{\mu_0 I}{2\pi d}\hat{x} = \frac{(4\pi\times10^{-7}\,\frac{T\,m}{A})(10\,A)}{2\pi\,(12\times10^{-2}\,m)}\hat{x} = 1.67\times10^{-5}\,T\,\hat{x}$$

↑ RH rule

(b) Give the x and y components of the magnetic field due to I_C at the location of I_A. What is the total magnetic field at the location of I_A? Indicate its direction graphically in the figure. (8 points)



$$\vec{B}_C = \frac{\mu_0 I}{2\pi(\sqrt{2}d)}\left(\frac{1}{\sqrt{2}}\hat{x} + \frac{1}{\sqrt{2}}\hat{y}\right) = \frac{\mu_0 I}{4\pi d}(\hat{x}+\hat{y}) = 8.3\times10^{-6}\,T(\hat{x}+\hat{y})$$

$$\vec{B}_{TOT} = \vec{B}_B + \vec{B}_C = \frac{\mu_0 I}{4\pi d}(3\hat{x}+\hat{y}) = (8.3\times10^{-6}\,T)(3\hat{x}+\hat{y})$$

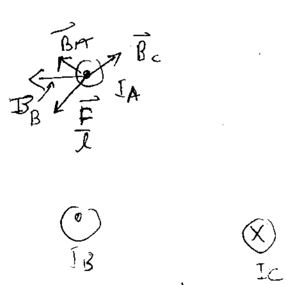
(c) What is the magnitude and direction of the force per unit length on I_A? (5 points)

$$\vec{F} = I\vec{l}\times\vec{B} \qquad \vec{l} = l\hat{z} \qquad \vec{B} = B_{TOT} = \frac{\mu_0 I}{4\pi d}(3\hat{x}+\hat{y})$$

$$\frac{\vec{F}}{l} = \frac{\mu_0 I^2}{4\pi d}(\hat{z}\times(3\hat{x}+\hat{y})) = \frac{\mu_0 I^2}{4\pi d}(3\hat{y}-\hat{x})$$

$$\boxed{\frac{\vec{F}}{l} = 8.3\times10^{-5}(3\hat{y}-\hat{x})\ N/m}$$

(d) How would the direction of the force per unit length on I_A change if the direction of I_B is reversed? (2 points)



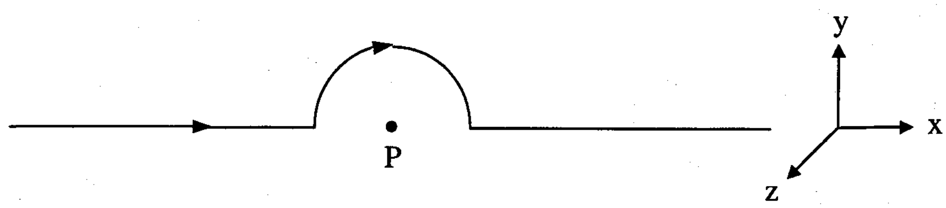
$$\vec{B}_B = \frac{\mu_0 I}{2\pi d}(-\hat{x})$$

$$\vec{B}_C = \frac{\mu_0 I}{4\pi d}(\hat{x}+\hat{y})$$

$$\vec{B}_{TOT} = \frac{\mu_0 I}{4\pi d}(-\hat{x}+\hat{y})$$

$$\frac{\vec{F}}{l} = I\frac{\vec{l}}{l}\times\vec{B} = \frac{\mu_0 I^2}{4\pi d}(\hat{z}\times(-\hat{x}+\hat{y})) = \frac{\mu_0 I^2}{4\pi d}(-\hat{x}-\hat{y})$$

5. A long wire (shown below) carries a current $I = 2$ A along its length. The semicircular arc has radius $R = 50$ cm.



(a) Find the magnetic field $\vec{B}$ at P. (10 points)

The B field at P due to the straight parts of the wire is zero
$(d\vec{l} \parallel \vec{r})$

∴ the field at P arises solely from the semicircular arc →

$B = \frac{\mu_0 I \theta}{4\pi R} = \frac{\mu_0 I \pi}{4\pi R} = \frac{\mu_0 I}{4R}$. Direction → $-z$ by RH rule.

$\boxed{\vec{B} = \frac{\mu_0 I}{4R}(-\hat{z})}$ → $\vec{B} = -\frac{(4\pi\times10^{-7}\frac{Tm}{A})(2\ A)}{4(.5\ m)}\hat{z} = -(4\pi\times10^{-7}\ T)\hat{z}$

(b) A magnet with magnetic dipole moment $\vec{\mu}$ ($|\vec{\mu}| = 1.5\times10^{-6}\ \mathrm{A\cdot m^2}$) oriented along the z axis is placed at P. What is the torque $\vec{\tau}$ on $\vec{\mu}$? Note: the size of the magnet is sufficiently small such that the magnetic field can be assumed to be constant over the length of the dipole. (5 points)

$\vec{\tau} = \vec{\mu}\times\vec{B}$

but $\vec{\mu} \parallel \vec{B}$ (actually antiparallel) since $\vec{\mu} = |\vec{\mu}|\hat{z}$

$\boxed{\vec{\tau} = 0}$

(c) If the magnet is given by a single loop of wire with a radius of 4 mm, what is the current in the loop? (5 points)

$\mu = I_l A = I_l \pi r^2$

$I_l = \frac{\mu}{\pi r^2} = \frac{(1.5\times10^{-6}\ \mathrm{A\,m^2})}{\pi(4\times10^{-3}\ \mathrm{m})^2} = \underline{0.03\ \mathrm{A}}$