Physics 202 Midterm Exam 1
October 1, 2007

Name: Key ID#:

Section:

TA (please circle):

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Instructions:

1. Don't forget to write down your name and section number.
2. **Show your work! A reasonable amount of work is required to receive full credit.**
3. Be aware that intermediate steps earn points even if the final answer is incorrect.
4. Erase (or cross out) any mistakes or you will be marked down. Grading is based on everything you have written down.
5. If you are asked for a vector quantity, both magnitude and direction need to be specified to be eligible for full credit.

Fundamental Constants:

$k_e = 8.99\times10^9\,\mathrm{N\cdot m^2/C^2}$ $\varepsilon_0 = (4\pi k_e)^{-1} = 8.85\times10^{-12}\,\mathrm{C^2/(N\cdot m^2)}$

$m_p = 1.67\times10^{-27}\,\mathrm{kg}$ $m_e = 9.11\times10^{-31}\,\mathrm{kg}$ $q_p = -q_e = 1.6\times10^{-19}\,\mathrm{C}$

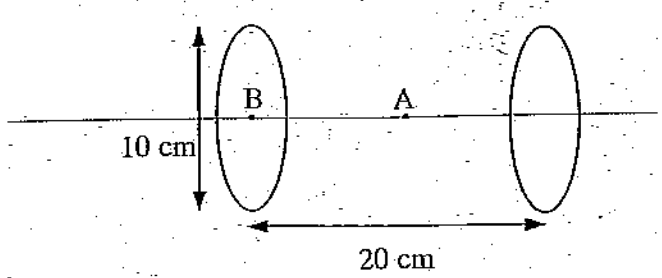
Scores:

Problem 1 ______________ **Problem 4** ______________

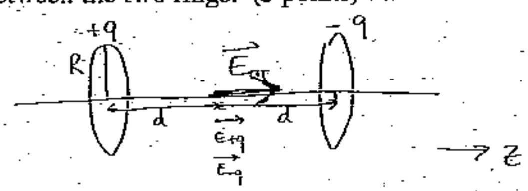
Problem 2 ______________ **Problem 5** ______________

Problem 3 ______________ **Problem 6** ______________

1. Two 10 cm diameter uniformly charged rings face each other, 20 cm apart. The left ring is charged to +15 nC and the right ring is charged to -15 nC.



(a) Based on the symmetry of the problem, indicate the direction of the electric field at the midpoint A on the axis between the two rings. (5 points)



(b) What is the electric potential at point A? (5 points)

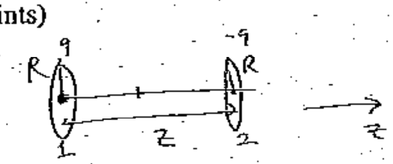
Call A the origin: then

$$V = \frac{k_e\, q}{\sqrt{R^2+d^2}} - \frac{k_e |q|}{\sqrt{R^2+d^2}} = 0$$

left ring — right ring

(c) What is the force $\vec{F}$ on a particle of charge $q_0 = -10$ nC located at the center of the left ring (point B)? (5 points)

Call point B the origin.



$z = .2\,m$

E field at center of ring 1 due to charge on ring 1 = 0 by symmetry

E field at center of ring 1 due to charge on ring 2 ⇒

$$\vec{E} = \frac{k_e |q| z}{(R^2+z^2)^{3/2}}\hat{z}$$

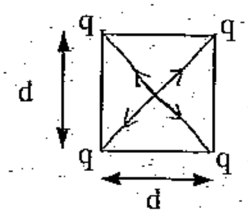
$$\vec{F} = q_0 \vec{E} = \frac{-|q_0||q| k_e z}{(z^2+R^2)^{3/2}}\hat{z}$$

$$= -\frac{(10\times10^{-9}C)(15\times10^{-9}C)(8.99\times10^{9}\ \frac{Nm^2}{C^2})(.2\,m)}{((.2)^2+(.05\,m)^2)^{3/2}}\hat{z}$$

$$= (-3.08\times10^{-5}\,N)\,\hat{z}$$

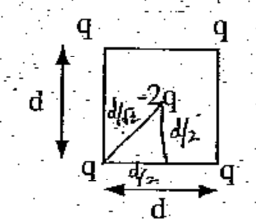
2. Four charges $q = 5\ \mu\text{C}$ sit at the corners of a square whose sides have length $d = 10$ cm.

(a) What is the electric field at the center of the square? (5 points)



By symmetry → $\vec{E} = 0$ at center of square

(b) What is the work that must be done by an external agent to move a negative charge $-2q = -10\ \mu\text{C}$ from infinitely far away to the center of the square? (Assume the other charges remain at fixed positions.) (10 points)



$$W = -2|q|\Delta V = -2|q|\, k_e \sum_i q_i/|r_i|$$

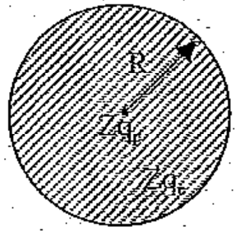
$$= -2|q|\, q\, k_e \frac{4}{(d/\sqrt{2})}$$

$$= -8 q^2 k_e \left(\frac{\sqrt{2}}{d}\right)$$

$$= \frac{-8\sqrt{2}\,(5\times10^{-6}\,C)^2\,(8.99\times10^{9}\ N\,m^2/C^2)}{(.1\,m)}$$

$$= -25.4\ J$$

3. An early model of the atom, proposed by Rutherford after his discovery of the atomic nucleus, had a positive point charge Zq_p (the nucleus) at the center of a sphere of radius R with uniformly distributed negative charge Zq_e. Z is the atomic number, the number of protons in the nucleus and the number of electrons in the negatively charged sphere.

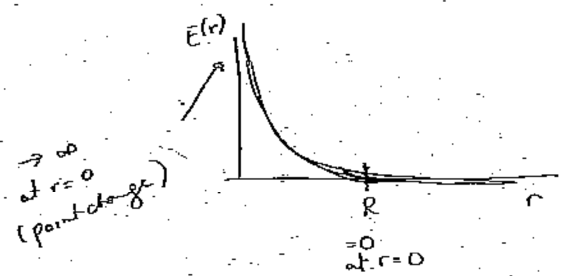


(a) Determine the electric field both inside $(r < R)$ and outside $(r > R)$ the atom. Sketch the behavior of the field as a function of r. (15 points)

$r<R$ $\quad \oint \vec{E}\cdot d\vec{A} = q_{in}/\epsilon_0$

$$E 4\pi r^2 = \frac{1}{\epsilon_0}\left(Zq_p + Zq_e \frac{\frac{4}{3}\pi r^3}{4/3\,\pi R^3}\right) \qquad \text{recall } q_e = -q_p$$

$$\Rightarrow \vec{E} = \left(\frac{Zq_p}{4\pi\epsilon_0 r^2} + \frac{Zq_e r}{4\pi\epsilon_0 R^3}\right)\hat{r} = k_e Z q_p\left(\frac{1}{r^2} - \frac{r}{R^3}\right)\hat{r} \quad \underline{\text{inside}}$$



$r>R$ $\quad \oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\epsilon_0} = 0$

$\vec{E} = 0$ outside

note → $\vec{E} = 0$ at $r = R$

(b) What is the electric potential $V(r)$ inside the atom? (Take the reference value where the potential vanishes at $r = \infty$.) (5 points)

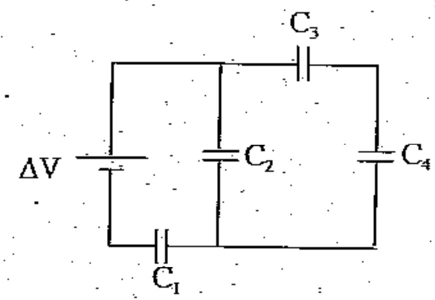
$$V(r) = -\int_{\infty}^{r} \vec{E}_r\, dr = -\int_{R}^{r} E_{in}\, dr \qquad \text{since } E_{out} = 0$$

$$= -k_e Z q_p \int_{R}^{r}\left(\frac{1}{r^2} - \frac{r}{R^3}\right) dr$$

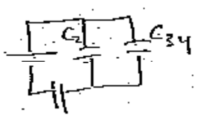
$$= -k_e Z q_p \left(\frac{-1}{r} - \frac{r^2}{2R^3}\right)\Bigg|_{R}^{r}$$

$$= k_e Z q_p \left(\frac{1}{r} + \frac{r^2}{2R^3} - \frac{3}{2R}\right)$$

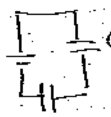
4. The following arrangement of parallel plate capacitors, with $C_1 = C_2 = C_3 = C_4 = 10\mu\text{F}$, is connected to a battery, which supplies a potential difference of 110 V.



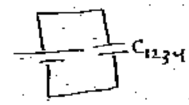
(a) What is the equivalent capacitance of this combination of capacitors? (5 points) let $C_1 = C_2 = C_3 = C_4 = C$



$C_{34} = \frac{C_3 C_4}{C_3 + C_4} = C/2$



$C_{234} = C_2 + C_{34} = 3C/2$



$C_{1234} = \frac{C_1 C_{234}}{C_1 + C_{234}} = \frac{C(3C/2)}{5C/2} = 3C/5$

$C_{eq} = C_{1234} = \frac{3(10\mu F)}{5} = 6\mu F$

$C_{eq} = 6\mu F$

(b) Which capacitor stores more charge, C_1 or C_3? Explain why. (5 points)

(C_1) → C_1 has charge Q of eq. cap. since in series w/ C_{234}. Charge splits in parallel branches → charge in 34 branch + charge in 2 branch = Q_{C1}.

Calc. not necessary if they argue properly

$Q = C_{eq} \Delta V = Q_{C1} = Q_{C234}$

$\Delta V_{C234} = \frac{Q_{C234}}{C_{234}} = \frac{Q_{C1}}{C_{234}}$

$Q_{34} = C_{34} \Delta V_{C234} = \frac{C_{34} Q_{C1}}{C_{234}}$

$Q_{34} = Q_3 \Rightarrow$ $Q_3/Q_1 = C_{34}/C_{234} = \frac{C/2}{3C/2} = C/3$

(C_1)

(c) The spacing of the plates of capacitor C_3 is now doubled (the other capacitors are unchanged). What is the new equivalent capacitance of the combination? (5 points)

$C_3 \to C_3' = C/2$

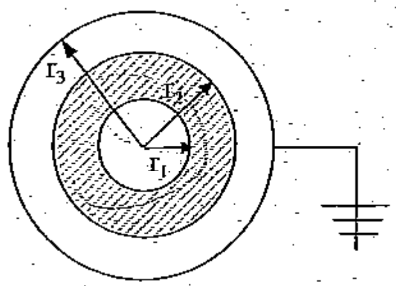
$C_{34}' = \frac{C_3' C_4}{C_3' + C_4} = \frac{C/2 \; C}{3C/2} = C/3$

$C_{234}' = C_2 + C_{34}' = 4C/3$

$C_{1234}' = \frac{C_1 C_{234}'}{C_1 + C_{234}'} = \frac{C \; 4C/3}{7C/3} = \frac{4}{7} C = \frac{4}{7}(10\mu F) = 5.7\mu F$

$C_{eq} = 5.7\mu F$

5. An infinitely long insulating cylindrical shell with inner radius r_1 and outer radius r_2 has a charge $+q$ uniformly distributed throughout its volume (with volume charge density ρ). It is surrounded by an infinitely long hollow cylindrical conductor with radius r_3, which is connected to ground $(V(r_3)=0)$. The arrangement is shown head-on in the figure below.



(a) What is the electric field $\vec{E}$ for $r < r_1$, $r_1 < r < r_2$, and $r_2 < r < r_3$? (15 points)

E in radial direction

$r < r_1$ $\quad \oint \vec{E} \cdot d\vec{A} = q_{in}/\epsilon_0 \rightarrow E = 0$ since $q_{in} = 0$

$\rho V = q$ (↳ volume)

$\rho \pi (r_2^2 - r_1^2) L = q$

$r_1 < r < r_2$ $\quad E \, 2\pi r L = \frac{1}{\epsilon_0} \rho \pi (r^2 - r_1^2) L$

$$\vec{E} = \frac{1}{2\epsilon_0} \rho \left(r - \frac{r_1^2}{r} \right) \hat{r}$$

$r_2 < r < r_3$ $\quad E \, 2\pi r L = \frac{1}{\epsilon_0} \rho \pi (r_2^2 - r_1^2) L$

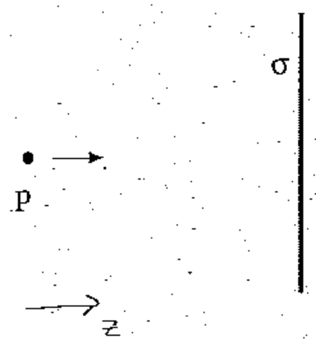
$$\vec{E} = \frac{\rho}{2\epsilon_0 r} (r_2^2 - r_1^2) \hat{r}$$

(b) What is the electric potential difference $V(r_2) - V(r_3)$? (5 points)

$$V = -\int_{r_3}^{r_2} E \, dr = -\int_{r_3}^{r_2} \rho \frac{(r_2^2 - r_1^2)}{2\epsilon_0 r} dr = -\frac{\rho (r_2^2 - r_1^2)}{2\epsilon_0} \ln r \Big|_{r_3}^{r_2}$$

$$V(r) = \frac{\rho (r_2^2 - r_1^2)}{2\epsilon_0} \ln r_3 / r_2$$

6. A proton with kinetic energy 2×10^6 eV (1 eV $= 1.6 \times 10^{-19}$ J) is fired in a direction perpendicular to the face of a large uniform sheet of charge with surface charge density $\sigma = 8.0 \times 10^{-6}$ C/m^2. (Assume the sheet is infinite in extent. Neglect any gravitational effects on the proton.)



(a) Determine the magnitude and direction of the proton's acceleration due to the electrostatic force exerted on it by the charged sheet. (5 points)

$$\vec{E} = \sigma/2\epsilon_0 (-\hat{z})$$

$$\vec{a}_p = \frac{q_p \vec{E}}{m_p} = \frac{-(1.6\times10^{-19}C)(8\times10^{-6}C/m^2)}{(1.67\times10^{-27}kg)2(8.85\times10^{-12}C^2/Nm^2)}\hat{z} = (-4.3\times10^{13} m/s^2)\hat{z}$$

(b) How much work must the electric field do on the proton to bring it to rest? (5 points)

$$W = \Delta KE$$
$$= -2\times10^6 eV = -3.2\times10^{-13} J$$

(c) From what distance should the proton be fired so that it stops right at the surface of the charged sheet? (5 points)

From cons. of energy

$$KE_i = q_p|\Delta V| \qquad |\Delta V| = Ed$$
$$= q_p E d$$
$$\Rightarrow d = \frac{KE_i}{q_p E} = \frac{(+3.2\times10^{-13} J)2(8.85\times10^{-12} C^2/Nm^2)}{(1.6\times10^{-19}C)(8\times10^{-6} C/m^2)} = \underline{4.4 m}$$