Generalized Capacitance in Bilayer Systems*

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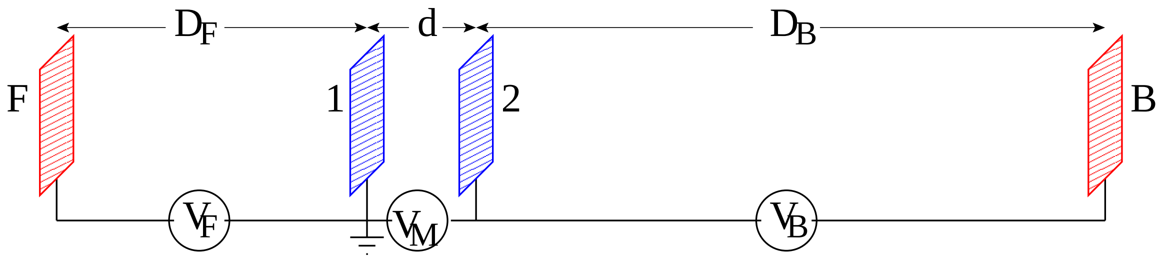
Double-Layer System: 2 Gates + 2 Layers



Typically, $\sqrt{A} \gg D_B \gg D_F \gg d$
(e.g., 1mm, 35μm, 0.3μm, 15nm)

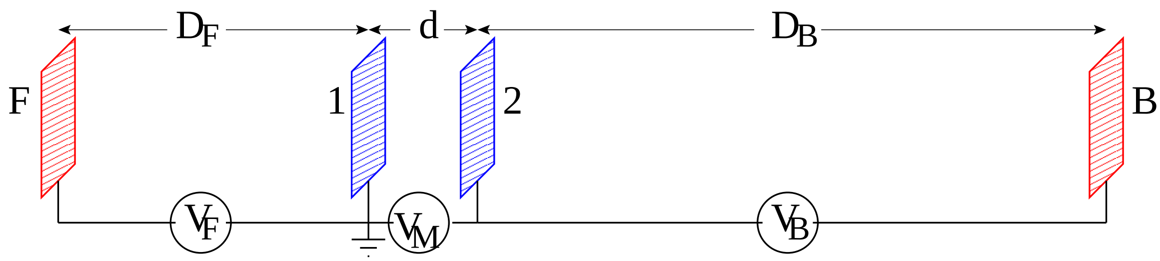
Goals: Model 2DE(H)G bilayer systems

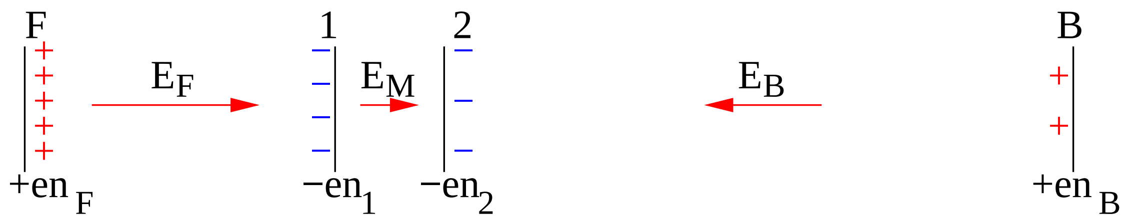
- Layer occupancies
- Capacitances
- Both B=0 and QH regime ($\nu_{\mathrm{T}} = 1$)

Include:

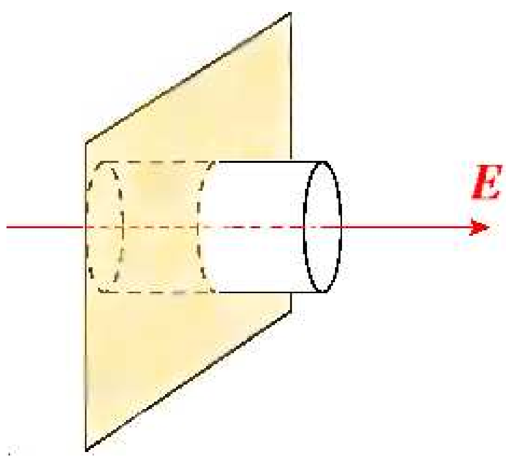
- Interlayer voltage, V_M
- Front and back gate
- Arbitrary dimensions D_B, D_F, d
- Calculate generalized capacitances

Electric Fields:





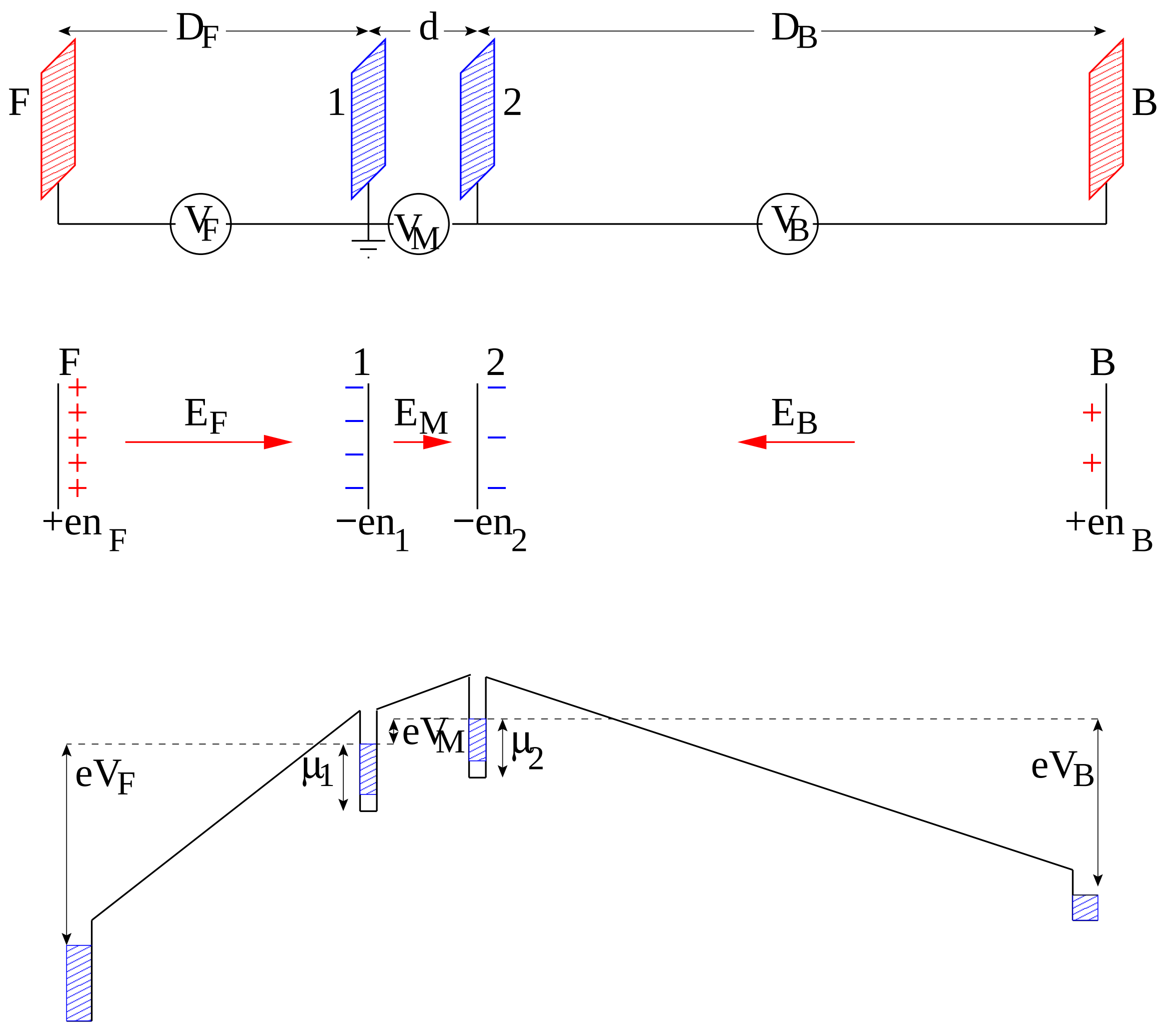
Gauss' Law:



$$|E\rangle = \begin{pmatrix} E_F \\ E_M \\ E_B \end{pmatrix} = \frac{1}{\epsilon} \begin{pmatrix} en_F \\ en_M \\ en_B \end{pmatrix}$$

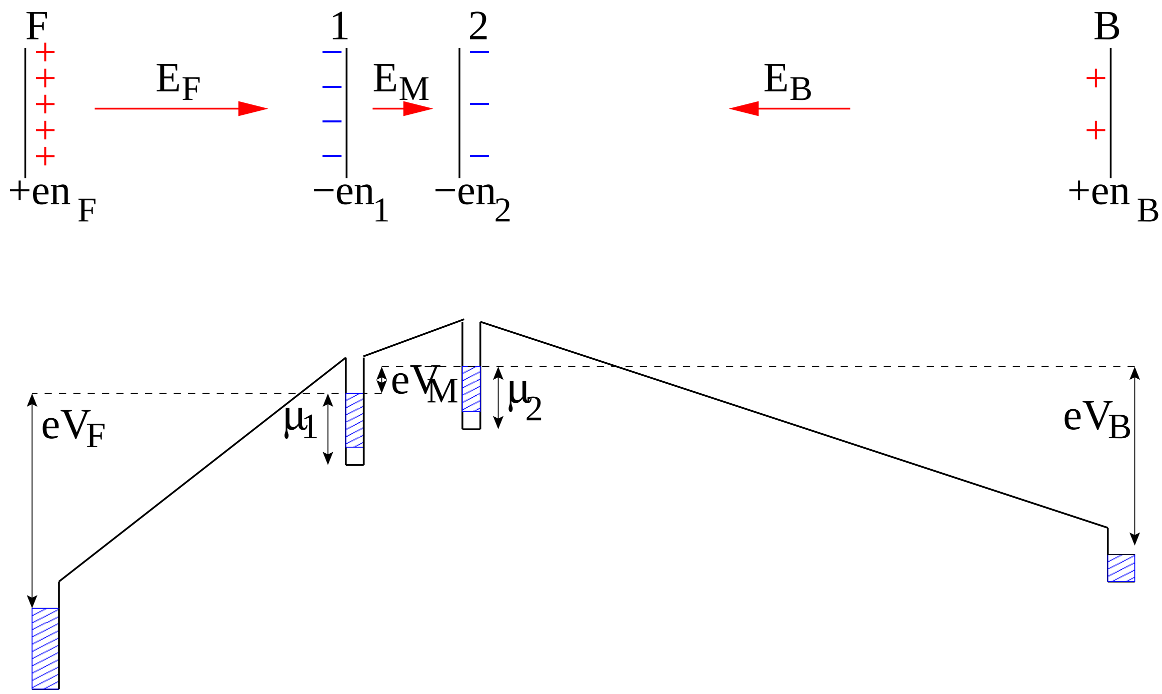
Charge neutrality: $n_M = n_F - n_1 = n_2 - n_B$

Voltages:



$$e|V\rangle = e\begin{pmatrix} V_F \\ V_M \\ V_B \end{pmatrix} = \begin{pmatrix} eE_F D_F + \mu_1 + eV_F^0 \\ eE_M d - \mu_1 + \mu_2 \\ eE_B D_B + \mu_2 + eV_B^0 \end{pmatrix}$$

Differential Capacitance Matrix:

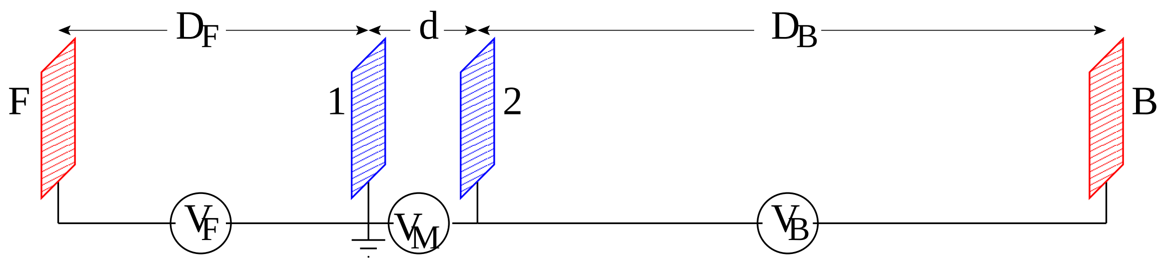


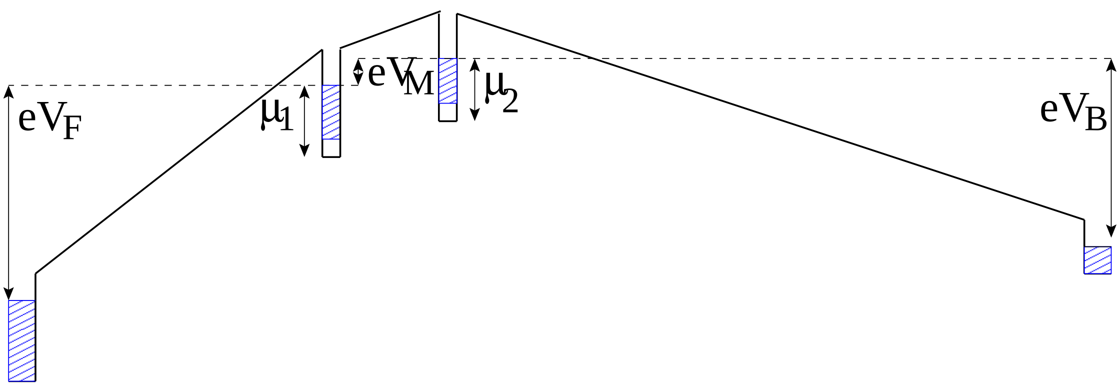
$$\mathbb{C} = \frac{|\delta Q\rangle}{|\delta V\rangle} = \epsilon \frac{|\delta E\rangle}{|\delta V\rangle} = \frac{\text{Capacitance}}{\text{Area}}$$

$$C_{\alpha\beta} = \langle \alpha | \mathbb{C} | \beta \rangle = \left. \frac{\delta Q_\alpha}{\delta V_\beta} \right|_{V_{\gamma \neq \beta}} ,$$

$$\alpha, \beta, \gamma = \mathrm{F},\ \mathrm{B},\ \mathrm{M} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Capacitive Lengths:





$$\mathbb{C} = \frac{\epsilon}{\mathcal{D}^3} \begin{bmatrix} D_B(d+s) + ds_{22} + S^2 & D_B(s_{11} - s_{12}) + S^2 & -ds_{12} - S^2 \\ D_B(s_{11} - s_{21}) + S^2 & D_B D_F + (D_F s_{22} + D_B s_{11}) + S^2 & -D_F(s_{22} - s_{12}) - S^2 \\ -ds_{21} - S^2 & -D_F(s_{22} - s_{21}) - S^2 & D_F(d+s) + ds_{11} + S^2 \end{bmatrix}$$

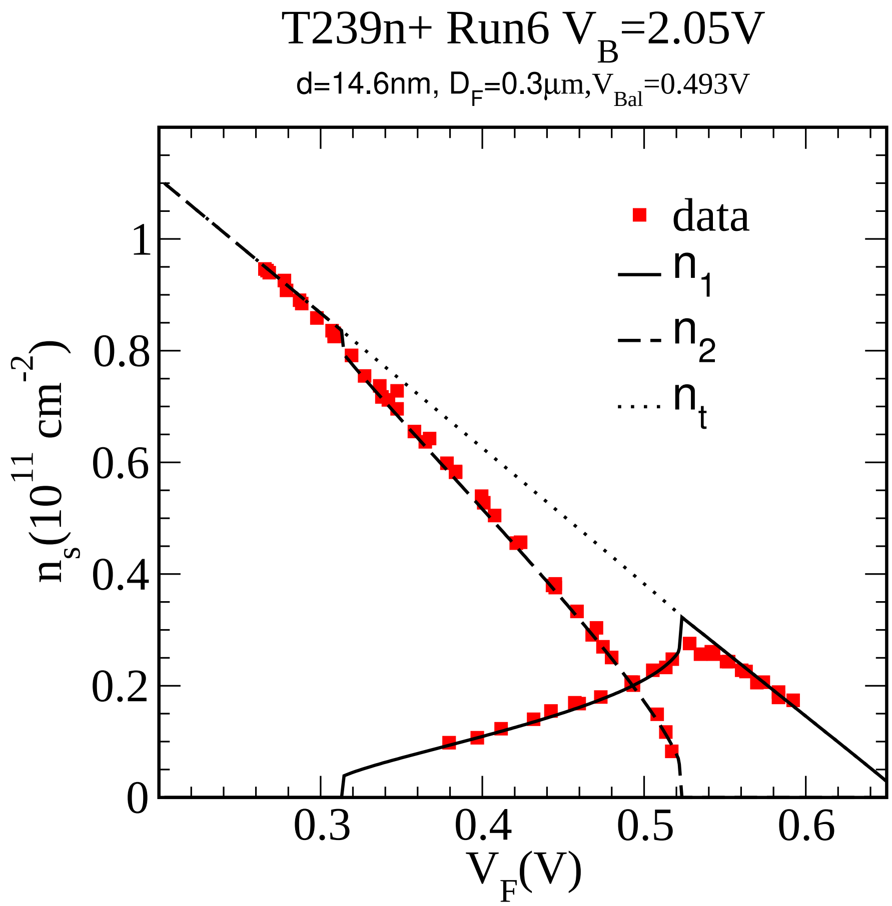
$$\mathcal{D}^3 = D_F D_B(d+s) + d(D_F s_{22} + D_B s_{11}) + (D_F + D_B + d)S^2 \ ,$$

$$S^2 = (s_{11}s_{22} - s_{12}s_{21}), \quad s = s_{11} - s_{21} + s_{22} - s_{12}$$

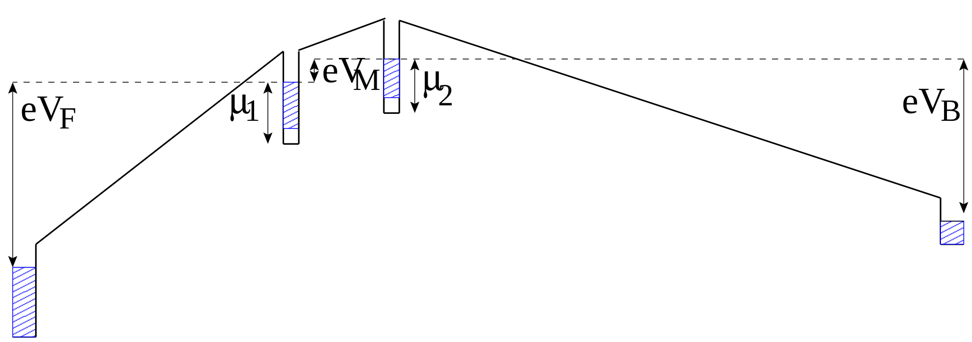
$$s_{ij} = \frac{\epsilon}{e^2}\frac{\partial \mu_i}{\partial n_j} = \frac{\partial^2}{\partial n_j \partial n_i}\varepsilon(n_1, n_2)$$

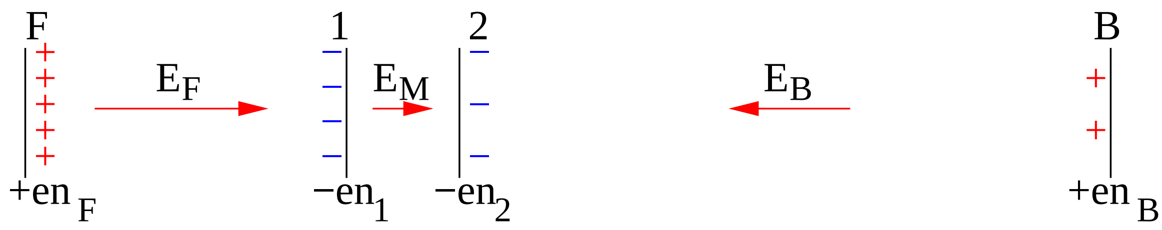
$$s_{ii} = \frac{\epsilon}{e^2 n_i^2}\frac{1}{\kappa_i}, \quad \kappa_i = n_i^2 \frac{\partial \varepsilon}{\partial n_i^2} < 0$$

SdH Data: Hamilton *et al.* (2002) "1-2-1"



Model: LDF + HFA energies, $s_{12} = s_{21} = 0$





Eisenstein Ratio: [PRL **68**, 674 (1992)]

…very sensitive measure of interlayer capacitance.

$$R_E(V_F) = \left.\frac{\delta E_M}{\delta E_F}\right|_{V_M,V_B} = \frac{C_{MF}}{C_{FF}} = 0 \text{ or } 1\ ?$$

$$= \frac{(s_{11}-s_{12}) + S^2/D_B}{(d+s) + ds_{22}/D_B + S^2/D_B} \approx \frac{s_{11}-s_{12}}{d+s}$$

$$\longrightarrow \frac{s_{11}}{d+s_{11}+s_{22}} \longrightarrow \frac{\epsilon}{e^2 d n_1^2}\frac{1}{\kappa_1}$$

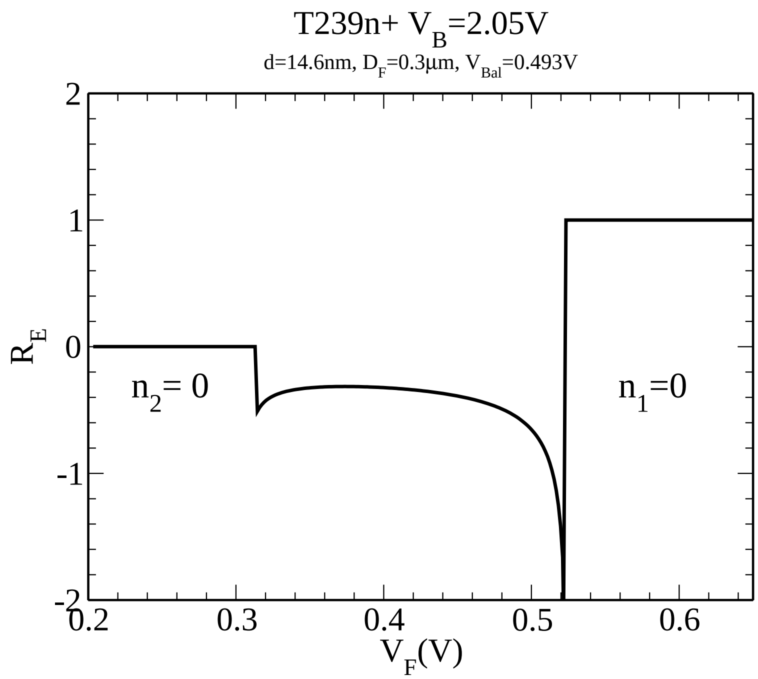
Back-Front Capacitance:

$$C_{BF} = \left.\frac{\delta Q_B}{\delta V_F}\right|_{V_M,\, V_B} = 0\ ?$$

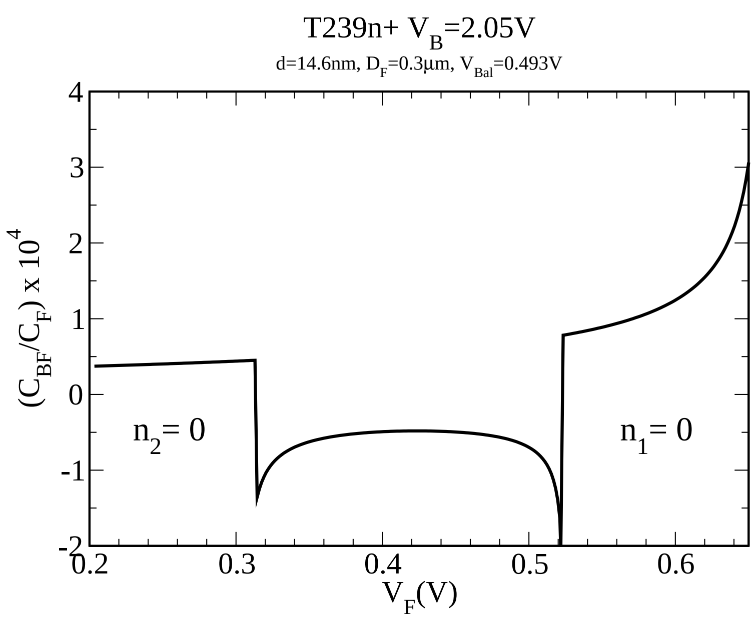
$$= -\frac{\epsilon(ds_{21}+S^2)}{\mathcal{D}^3} \longrightarrow -\epsilon\frac{s_{11}s_{22}}{D_F D_B(d+s_{11}+s_{22})}$$

$$\longrightarrow -\frac{\epsilon}{D_F D_B}\left(\frac{\epsilon}{e^2 n_1 n_2}\right)^2 \frac{1}{\kappa_1\kappa_2}$$

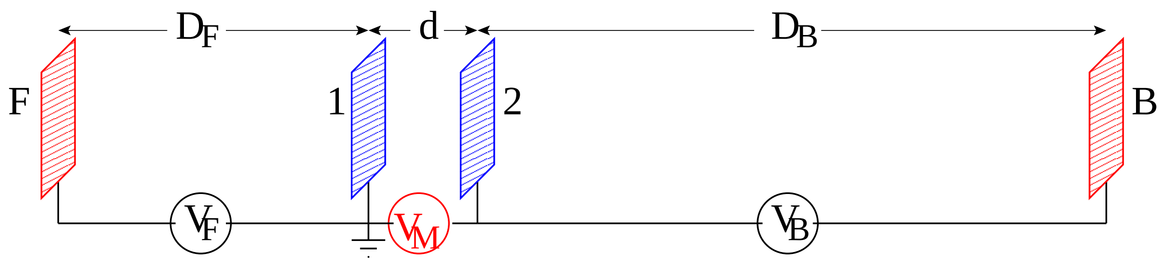
Eisenstein Ratio: $R_E(V_F) = \left.\frac{\delta E_M}{\delta E_F}\right|_{V_M,V_B} = \frac{C_{MF}}{C_{FF}}$



Back-Front Capacitance: $C_{BF} = \left.\frac{\delta Q_B}{\delta V_F}\right|_{V_M,V_B}$

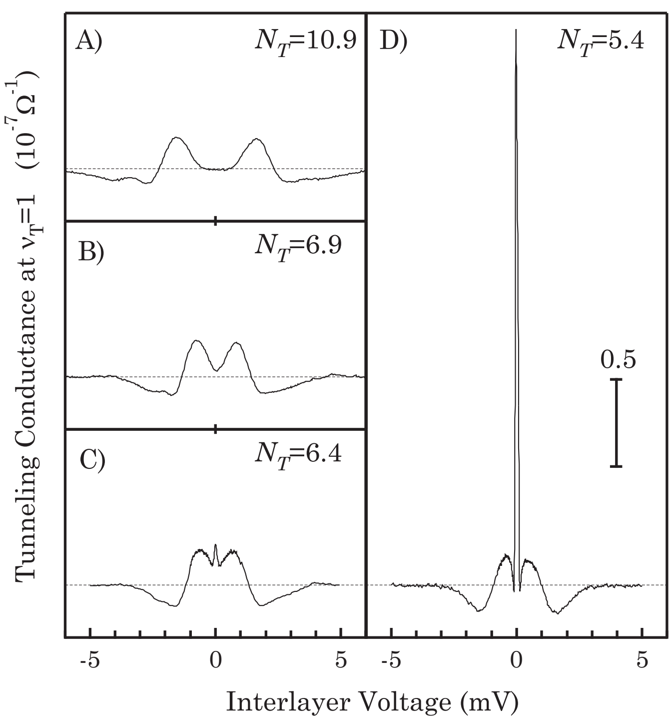


Separately Contactable Layers: $V_M \neq 0$



Eisenstein *et al.*

- PRL **88**, 126804 (2002)
- PRL **87**, 036803 (2001)
- PRL **84**, 5808 (2000)

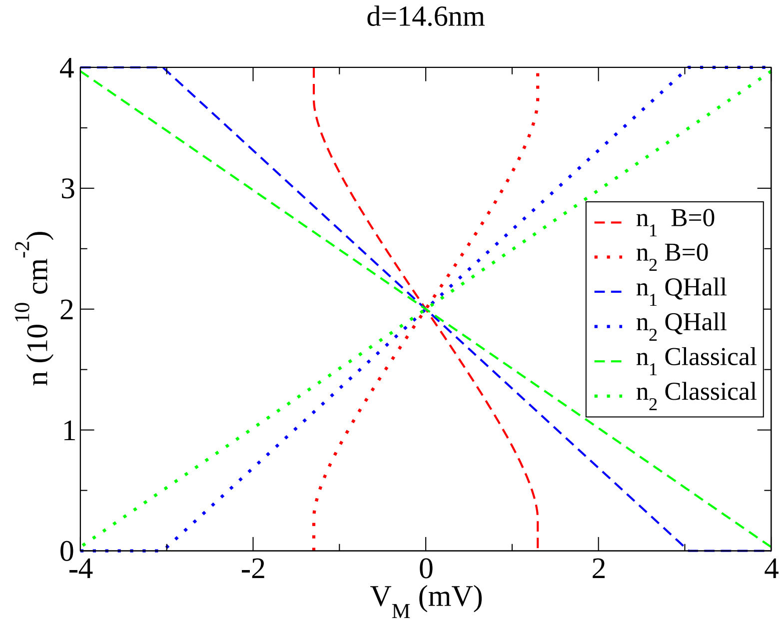


Quantum Hall Regime:

- Total filling factor $\nu_{\mathrm{T}} = 1$
- Small d/ℓ
- Interlayer exchange

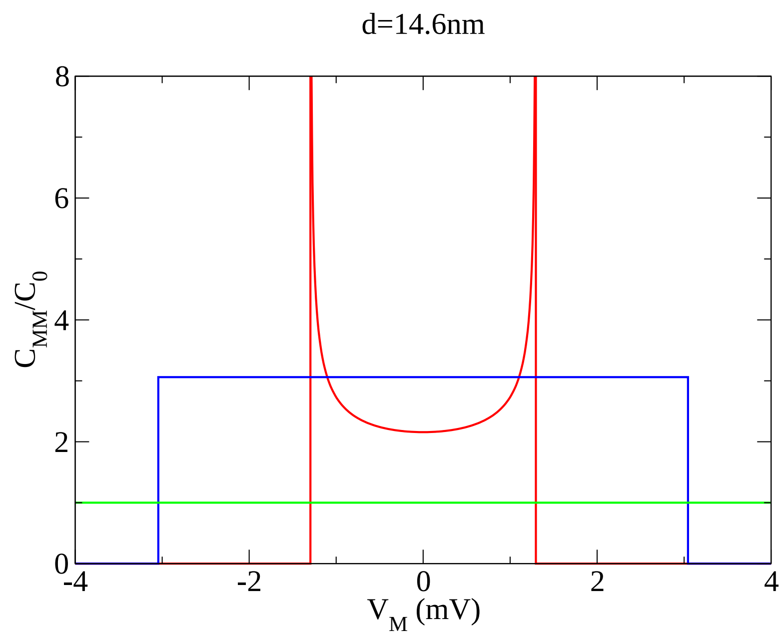
J. C. Díaz-Vélez, APS March 2003

Layer Densities vs. Interlayer V_M



Interlayer Capacitance:

$$C_{MM} = \frac{\delta Q_M}{\delta V_M}\bigg|_{V_F, V_B}$$



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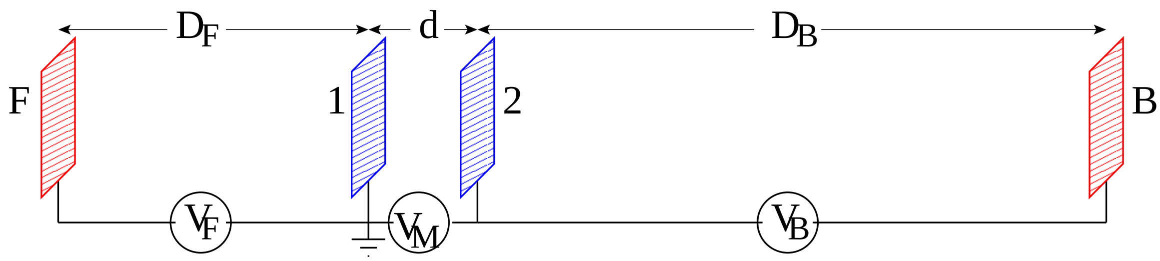
Summary:

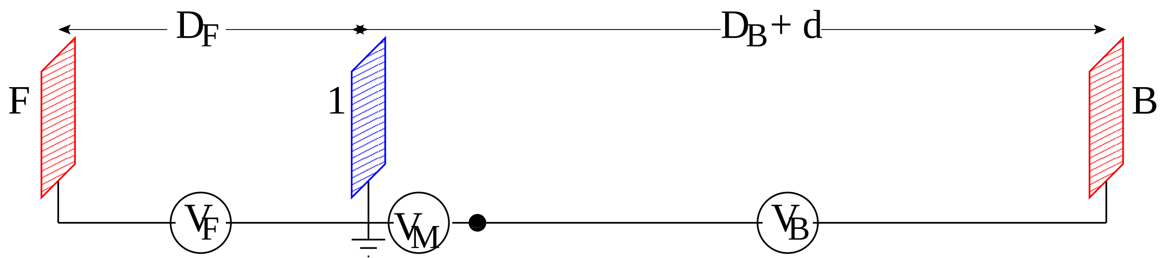


- 2 Gates (B,F) and 2 Layers (1,2)
- Arbitrary distances $D_F,\ d,\ D_B$
- Voltages V_F and V_B plus V_M
- Model calculations of $n_1,\ n_2,\ \kappa$
  ... Fit SdH data for "1-2-1" transition
- Derived generalized capacitance

$$C_{\alpha\beta} = \left.\frac{\delta Q_\alpha}{\delta V_\beta}\right|_{V_{\gamma\neq\beta}}$$

- Calculated R_E and C_{BF} for $B = 0$
- Calculated C_{MM} for B=0 and QH regime

One Empty Layer: $(n_2{=}0)$



$$e|V\rangle = e\begin{pmatrix} V_F \\ V_B - V_M \end{pmatrix} = \begin{pmatrix} eE_F D_F + \mu_1 + eV_F^0 \\ eE_B(D_B + d) + \mu_1 + eV_B^0 \end{pmatrix}$$

$$C_{MM} = \left.\frac{e\delta n_M}{\delta V_M}\right|_{V_F, V_B} = \left.\frac{-e\delta n_B}{-\delta V_B}\right|_{V_F, V_M} = C_{BB}$$

$$\sim \frac{\epsilon}{D_B + d} \ll \frac{\epsilon}{d}$$