Testing Lorentz Invariance using Atmospheric Neutrinos and AMANDA-II

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Abstract

Several phenomenological models of physics beyond the Standard Model predict flavor mixing in the neutrino sector in addition to conventional mass-induced oscillations. In particular, violation of Lorentz invariance (VLI) results in neutrino oscillation effects parametrized by the maximal attainable velocity difference $\delta c/c$. We report on a study of the sensitivity of the AMANDA-II detector to such effects using distortions in the spectrum of high-energy atmospheric neutrinos. For maximal mixing and six years of simulated data, the preliminary sensitivity of AMANDA-II to such VLI effects is $\delta c/c < 2.1 \times 10^{-27}$ at the 90% confidence level.

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1. Introduction

Flavor oscillations in the neutrino sector provide an interesting method to test phenomenological models of physics beyond the Standard Model. While mass-induced oscillations of atmospheric neutrinos are on firm experimental footing [1–3], subdominant effects may yet be present. In particular, violation of Lorentz invariance (VLI) can result in oscillations at high energies and distort the atmospheric neutrino spectrum.

The AMANDA-II detector, a subdetector of the IceCube experiment, is an array of 677 optical modules buried in the ice at the geographic South Pole which detects the Čerenkov radiation from charged particles produced in neutrino interactions with matter [4]. In particular, muons produced in charged-current ν_μ and $\bar{\nu}_\mu$ interactions deposit light in the detector with a track-like topology, allowing us to use directional reconstruction to reject the large background of down-going atmospheric muon events. After suitable quality selection criteria are applied, AMANDA-II accumulates atmospheric neutrino candidates above 50 GeV at a rate of ≈ 4 per day [5]. While conventional oscillations are suppressed at these energies, VLI effects can be detected or constrained by their influence on the zenith angle distribution and energy-correlated observables.

2. Phenomenology

Various new physics scenarios can result in neutrino flavor mixing beyond conventional oscillations. We focus here on oscillations induced by differing maximally attainable velocities (MAVs) in the neutrino sector. MAV eigenstates can be distinct from flavor eigenstates, resulting in oscillations characterized by the MAV difference $\delta c/c = (c_1 - c_2)/c$.

Conventional and VLI oscillations can be combined in a two-family scenario, with the following survival muon neutrino survival probability as a function of energy E and baseline L (in energy units) [6–8]:

$$P_{\nu_\mu \to \nu_\mu} = 1 \; - \; \sin^2 2\Theta \; \sin^2 \left(\frac{\Delta m^2 L}{4E} \; \mathcal{R} \right) \; , \tag{1}$$

where

$$\sin^2 2\Theta = \frac{1}{\mathcal{R}^2} (\sin^2 2\theta + R^2 \sin^2 2\xi + 2R \sin 2\theta \sin 2\xi \cos \eta) \; , \tag{2}$$

$$\mathcal{R} = \sqrt{1 + R^2 + 2R(\cos 2\theta \cos 2\xi + \sin 2\theta \sin 2\xi \cos \eta)} \; , \tag{3}$$

and

$$R = \frac{\delta c}{c} \frac{E}{2} \frac{4E}{\Delta m^2} \; . \tag{4}$$

Standard oscillations are characterized by the mass-squared difference Δm^2 and mixing angle θ, while the VLI oscillation parameters include the velocity difference $\delta c/c$, the mixing angle ξ, and the phase η. If we take both conventional and VLI mixing to be maximal ($\theta = \xi = \pi/4$) and set $\cos \eta = 1$, this reduces to the following:

$$P_{\nu_\mu \to \nu_\mu}(\text{maximal}) = 1 \; - \; \sin^2 \left(\frac{\Delta m^2 L}{4E} + \frac{\delta c}{c} \frac{LE}{2} \right) \; . \tag{5}$$

Note the different energy dependence of the two effects. For atmospheric neutrinos, the zenith angle functions as a surrogate for the baseline L, allowing path lengths up to the diameter of the Earth. Figure 1 shows the survival probability as a function of neutrino energy and zenith angle for the maximal case, as in equation 5.

3. Analysis Methodology

In general, to test the values of some parameters θ_i of our hypothesis (in the simplest one-dimensional case, just $\delta c/c$) that are allowed or excluded at some confidence level, we follow the likelihood prescription described by Feldman and Cousins [9]:

- For each point in the parameter space θ_i, we sample many times from the parent Monte Carlo distributions of the observable(s) (MC "experiments").

- For each MC experiment, we calculate the likelihood ratio $\Delta\mathcal{L} = \mathcal{L}_i - \mathcal{L}_{i,best}$, where $\mathcal{L}_i$ is the Poisson log-likelihood that the MC experiment is derived from the parent distribution at θ_i (other likelihood formulations are possible).

- For each point θ_i, we find the value $\Delta\mathcal{L}_{crit}$ at which, say, 90% of MC experiments have a lower $\Delta\mathcal{L}$.

- Finally, we compare the $\Delta\mathcal{L}$ of the data (or in our case, a simulated data set generated under the null hypothesis) with the critical surface $\Delta\mathcal{L}_{crit}$, and regions of the parameter space at which $\Delta\mathcal{L} > \Delta\mathcal{L}_{crit}$ are excluded at that confidence level. For a one-dimensional parameter space, this can likely be interpreted an upper limit, and one can calculate a median sensitivity by iterating over a number of

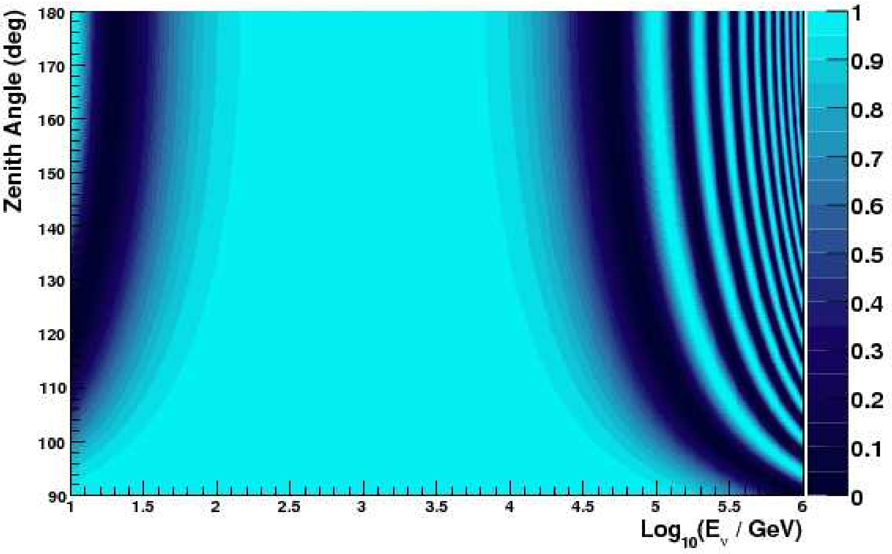

Fig. 1. Atmospheric ν_μ survival probability as function of neutrino energy and zenith angle. Conventional oscillations are present at low energies, while high-energy oscillations are due to VLI (maximal mixing, $\delta c/c = 10^{-27}$).

simulated data sets.

With perfect angular resolution and energy reconstruction, we would simply be able to map out the survival probability shown in Figure 1: even after folding in the detector response, the minima are obvious. While AMANDA-II has an angular resolution of a few degrees [10], reconstruction of the neutrino energy is more difficult and fundamentally limited by the stochastic losses of the muon. Instead, we use a well-simulated energy-correlated observable, the number of triggered optical modules (N_{ch}).

To obtain a clean sample of atmospheric neutrinos, we must separate these from the large background of atmospheric muons. Selecting events with a reconstructed zenith angle below the horizon allows rejection of many such events, but we must generally apply further quality criteria to eliminate mis-reconstructed muons. For this study, we have used the selection criteria from the 2000-03 AMANDA-II point source search [5] and examine only zenith angles $> 100^o$.

4. Sensitivity of AMANDA-II

We have performed a Monte Carlo study using six years of simulated AMANDA-II data: an integrated exposure of 1200 days, approximately 5100 events below the horizon under the null hypothesis (conventional oscillations only). For this initial study, we have tested only the N_{ch} distribution across a one-dimensional parameter space, varying the VLI strength $\delta c/c$. To anticipate the impact of the inclusion of systematic errors in the future, we have left free the normalization of the atmospheric neutrino flux (*i.e.* treating it as a nuisance parameter). The curves of $\Delta\mathcal{L}_{crit}$ for the 90%, 95%, and 99% confidence levels are shown in Figure 2, along with the likelihood ratio for a single simulated data set.

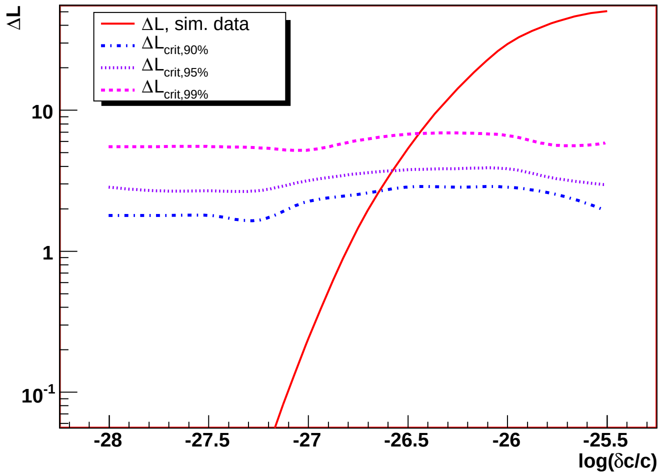

Fig. 2. Likelihood ratio for VLI effects using the shape of the N_{ch} distribution, for values of the parameter $\delta c/c$. The critical curves for various confidence levels are shown, along with $\Delta\mathcal{L}$ for a simulated six-year data set. Values of $\delta c/c$ to the right of the point of intersection with the critical curve are excluded.

Assuming maximal mixing ($\sin 2\xi = 1$) and phase $\cos\eta = 1$, we find a median sensitivity of $\delta c/c < 2.1 \times 10^{-27}$ at the 90% confidence level. Existing experimental limits include the MACRO result of $\delta c/c < 2.5 \times 10^{-26}$ [11] and the limit by González-García and Maltoni using the Super-Kamiokande + K2K data, $\delta c/c < 2.0 \times 10^{-27}$ [8].

5. Conclusions and Outlook

Using its large sample of atmospheric neutrinos, AMANDA-II is capable of detecting or constraining high-energy new physics effects in the neutrino sector. The Monte Carlo study presented here indicates a sensitivity to VLI effects competitive with existing limits, and a number of improvements (such as testing multiple observables) and optimizations (*e.g.* event selection criteria, and the binning of the observables) are forthcoming. We anticipate applying this analysis in the near future to the AMANDA-II data collected from 2000-2005.

Furthermore, the same methodology can also be applied to constrain other physics beyond the Standard Model, such as violations of the equivalence principle [13], or quantum decoherence resulting from interactions of neutrinos with the background space-time foam [14,15].

The next-generation IceCube detector, with an instrumented volume of 1 km^3, will provide unprecedented sensitivity to these same effects. In 10 years of operation, IceCube will collect a sample of more than 700 thousand atmospheric neutrinos above 100 GeV and will be sensitive at the 90% confidence level to VLI effects at the level of $\delta c/c < 2.0 \times 10^{-28}$ [12]. This high-statistics sample will also provide an opportunity to test other phenomenological models of physics beyond the Standard Model.

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