Washington University in St. Louis

McDonnell Center for the Space Sciences

Use of Stokes Parameters in Hard X-Ray Polarimetry

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Senior Honors Thesis Presentation

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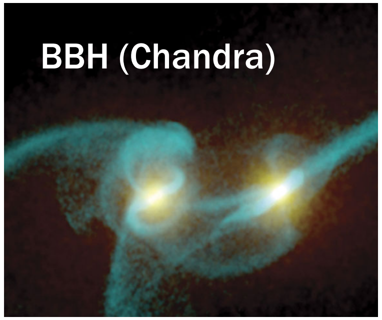
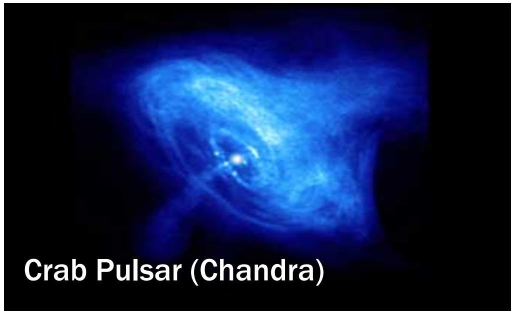
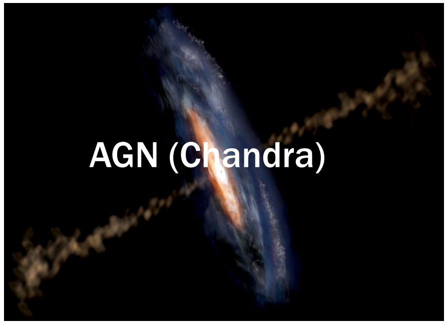
X-Ray and Gamma Ray Astroparticle Physics Group

Scientific Motivation

- **Binary Black Holes (Schnittman & Krolik 2010)**
  - Probe accretion disk inclination and spin
  - The shape of the x-ray corona
- **Pulsars**
  - Discriminate between physical processes—curvature vs. synchrotron emissions (different phase morphology)
  - Constrain/ determine shape of pulsar beam (oscillations in amount of x-ray polarization shape dependent)
- **Active Galactic Nuclei (AGN) and their Jets (Krawczynski 2012)**
  - Information about magnetic field near jet base (hard x-ray emission in uniform field near base)



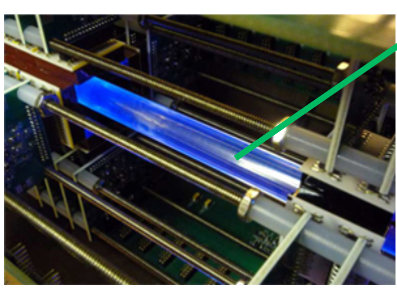


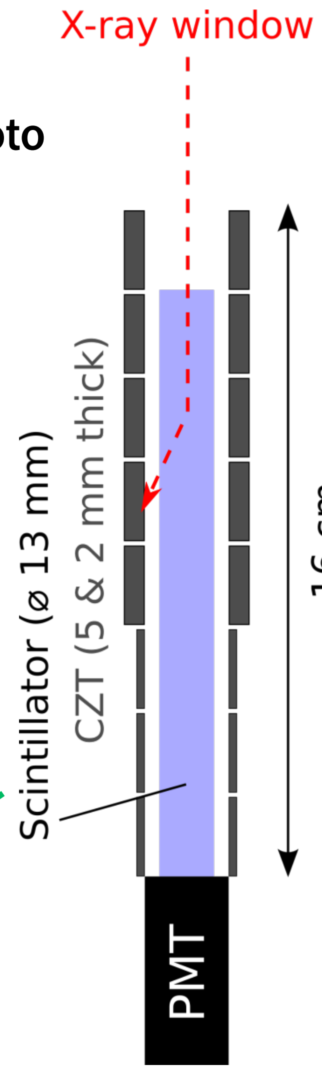


X-Calibur Principles

Photons interact with materials by Compton scattering (low-Z) or photo absorption (hi-Z).

X-Calibur combines both. Low-z Scintillator Compton scatters the incident photon; hi-Z CZT detector photo-absorbs it.





This scattering is governed by the Klein-Nishina Cross Section

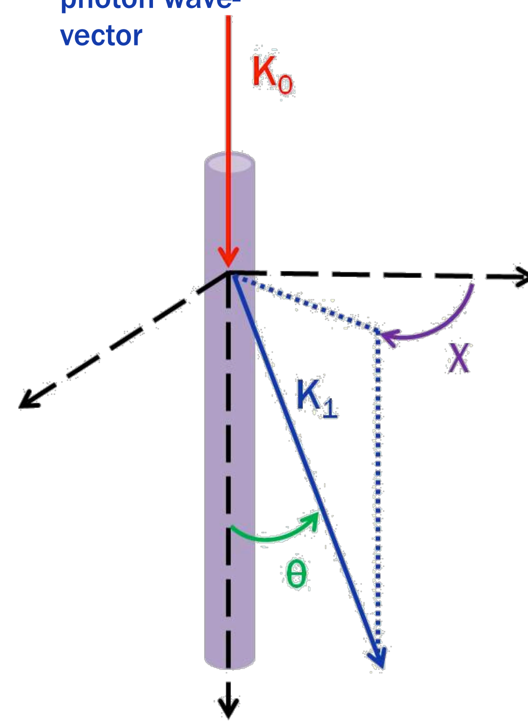
$$\frac{d\sigma}{d\Omega}(\theta) = \frac{r_0^2}{2}\frac{k_1^2}{k_0^2}\left[\frac{k_0}{k_1} + \frac{k_1}{k_0} - 2\sin^2\theta\cos^2\chi\right]$$

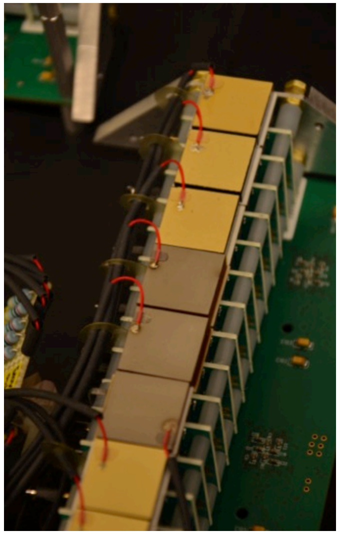
Incident photon wave-vector

Scattered photon wave-vector

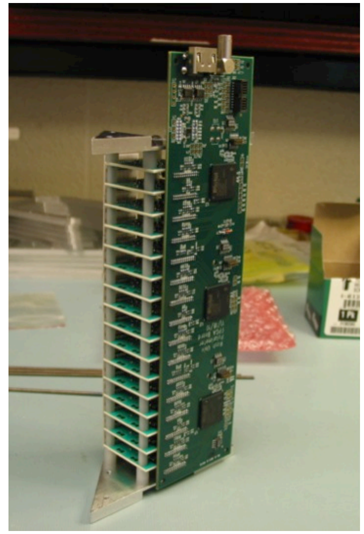
Angle between k_0 and k_1

Angle between E-field vector and scattering plane. (azimuthal angle)

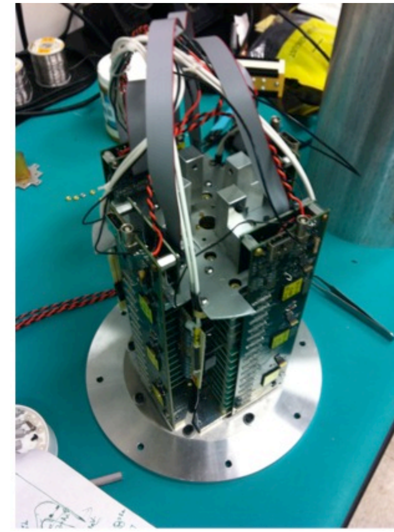




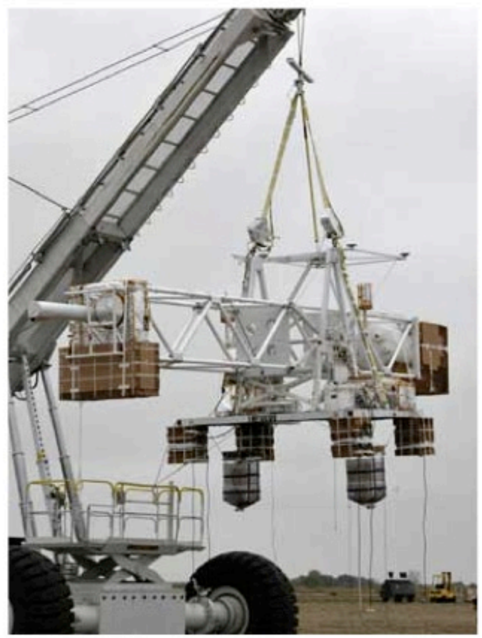
CZT detector panel



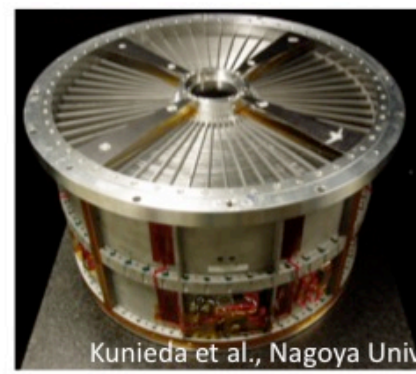
ASIC board (read-out CZTs)



4 ASIC boards around the scintillator (CZTs face inwards)



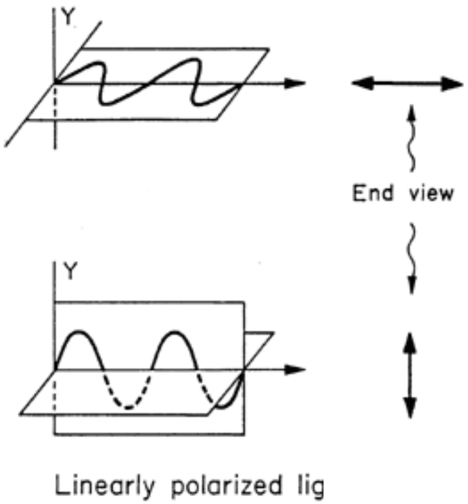
InFOCμS gondola with 8m optical bench
(Tueller et al., NASA GSFC)



255 shell Al mirror with $50\ cm^2$ area at 30 keV (Pt/C coating).

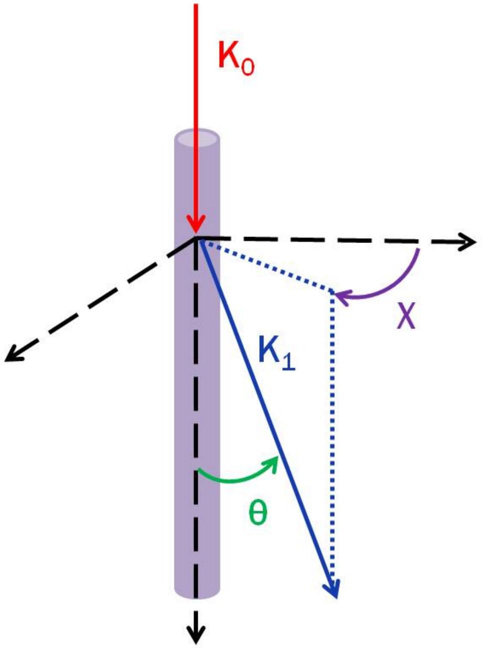
Characterizing Polarization

- Polarization—oscillations in the plane perpendicular to a wave's direction of travel
- Can be linearly or elliptically polarized
  - Cannot measure elliptical polarization with apparatus like X-Calibur
  - That's ok, galactic and extragalactic objects exhibit linear polarization mostly, anyway (Krawczysnki 2012)



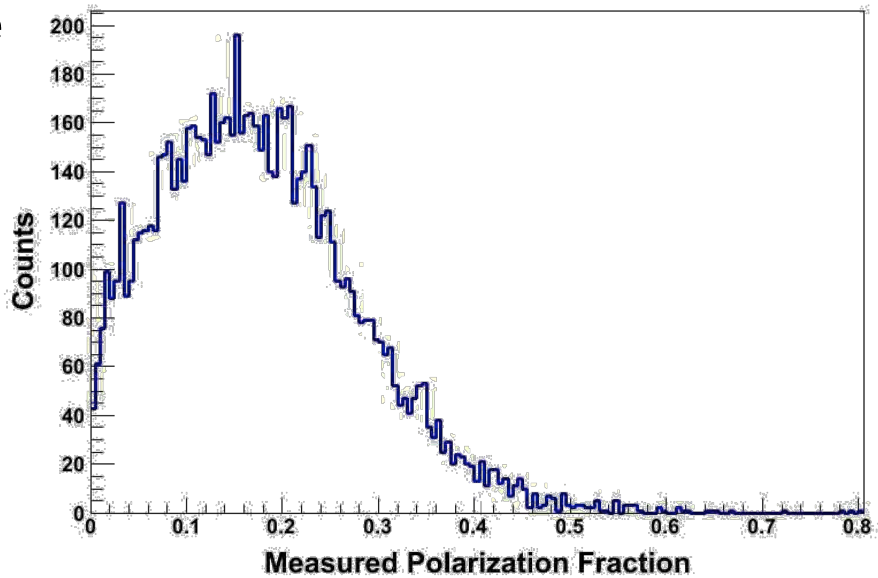
- Linear polarization characterized by polarization angle and fraction

- <u>Fraction (π)</u> —portion of the wave that is polarized
- <u>Angle (χ)</u> —the angle of the projection of the polarization vector relative to the vertical axis (in the case of polarimetry)
- <u>Modulation factor (μ)</u> –defined as the observed polarization fraction for a 100% polarized beam
  - Dependent on physics of Compton scattering and design of polarimeter
  - For X-Calibur, μ~0.5



My Project

- Astrophysical source has some unknown polarization (π_{true}, χ_{true})
- We have limited experimental statistics (cannot observe the source forever!)
- Even in a beam with 0 polarization fraction, always measure *something;* $\pi_{computed}$ must be ≥0
- Therefore, for a small π_{true}, distribution of measured fraction is not centered about the true value



- Using Bayesian statistics:
  - Find better guess (π_{guess}) of π_{true}
  - Create probability distribution for this π_{guess}
  - Use the probability distribution to set error bars on a π_{guess} given μ

Analysis Techniques—Stokes Parameters

- Unique to my project: propose analyze the signal photon-by-photon, using a Stokes decomposition
- Decompose wave into its intensity, horizontal, vertical, and circular polarization components
  - We only work with the first three

For single photon, have azimuthal scattering angle (χ), define q and u (the horizontal and vertical components of polarizaton):

$$q = \cos(2\chi) \quad u = \sin(2\chi) \quad i = n$$

For a collection of photons, define fractional degree of polarization (π_0) and polarization angle (χ_0):

$$\pi_0 = \frac{2}{\pi}\frac{\sqrt{Q^2 + U^2}}{I} \quad \tan(2\chi_0) = \frac{U}{Q}$$

$$Q = \sum_j q_j \quad U = \sum_j u_j \quad I = \sum_j i_j = N$$

Analysis Techniques—Statistics

- I simulate a source with (π_{true}, χ_{true}).
- Model the detection of these photons in a generic detector with modulation factor μ
- Compute best guess for true polarization, call it (π_{guess}, χ_{guess}).

The probability of measuring a random (Q,U) given we know the true values (Q_0,U_0).

$$P(Q,U \mid Q_0,U_0) = \underbrace{P(Q \mid Q_0)P(U \mid U_0)}_{\text{Independent variables}}$$

$$= \frac{1}{2\pi\sigma^2}\exp\left[-\frac{(Q-Q_0)^2+(U-U_0)^2}{2\sigma^2}\right]$$

Convert from intensities to polarization fraction and angle:

$$P(\pi,\chi \mid \pi_0,\chi_0) =$$

$$= \frac{1}{2\pi\sigma^2}\exp\left[-\frac{\pi_0^2+\pi^2-2\cos[2(\chi-\chi_0)]}{2\sigma^2}\right]$$

Analysis Techniques—Statistics

This is P(π, χ | π_0, χ_0).
Want P(π_0, χ_0 | π, χ).

Apply Bayes' Theorem to "invert" the distribution.

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Applied to our system:

$$P(\pi_0,\chi_0|\pi,\chi) = \frac{P(\pi,\chi|\pi_0,\chi_0)P(\pi_0,\chi_0)}{\int_0^1\int_0^\pi d\pi_0 d\chi_0\left[P(\pi,\chi|\pi_0,\chi_0)P(\pi_0,\chi_0)\right]}$$

The prior (what we assume we know about the polarization of the signal). For me, =1 (know nothing, worst case scenario)

$$P(\pi_0,\chi_0)$$

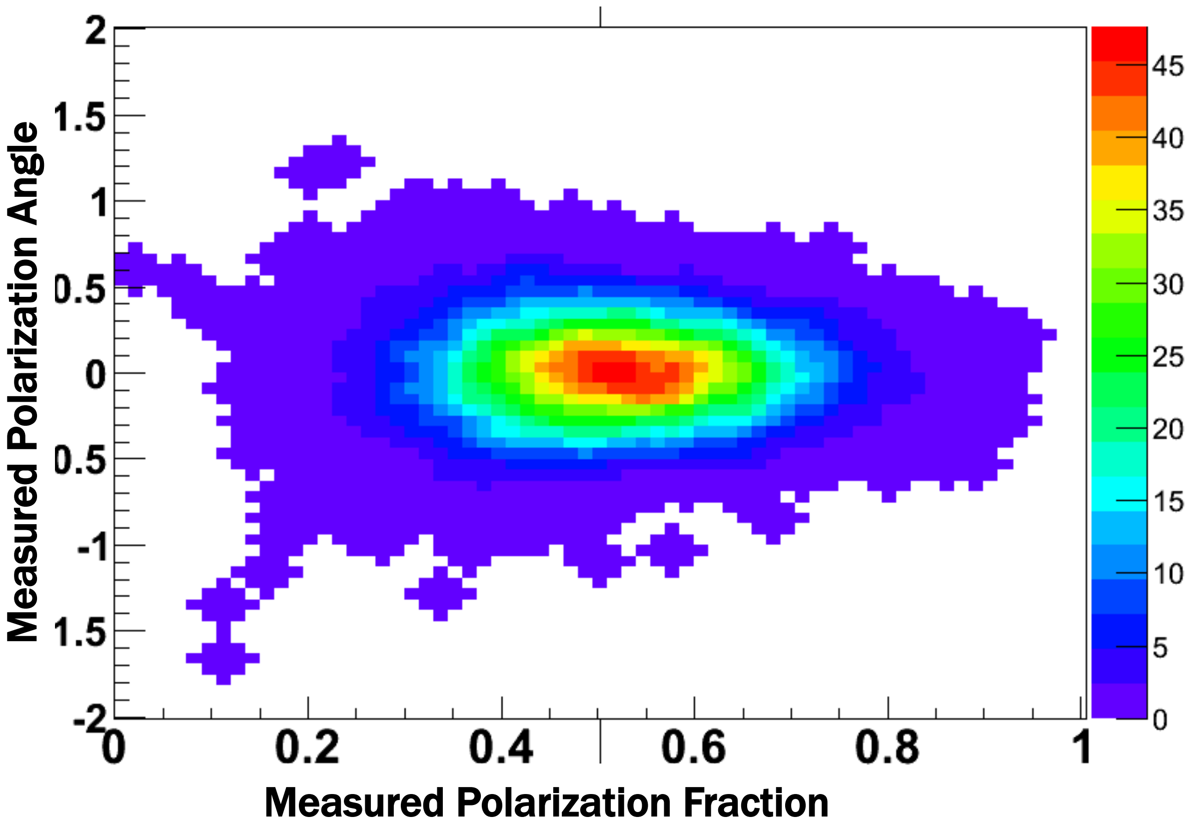
Methods

Steps 1→3: Simulating Measurement Runs

1- Generation of photons for one fixed (π_0,χ_0), and given μ, according to Klein-Nishina.

2-Extraction of polarization parameters (Q,U,π,χ) for each measurement run.

3- Simulate many measurement runs (repeat 1-2), plot the distribution of (π,χ)



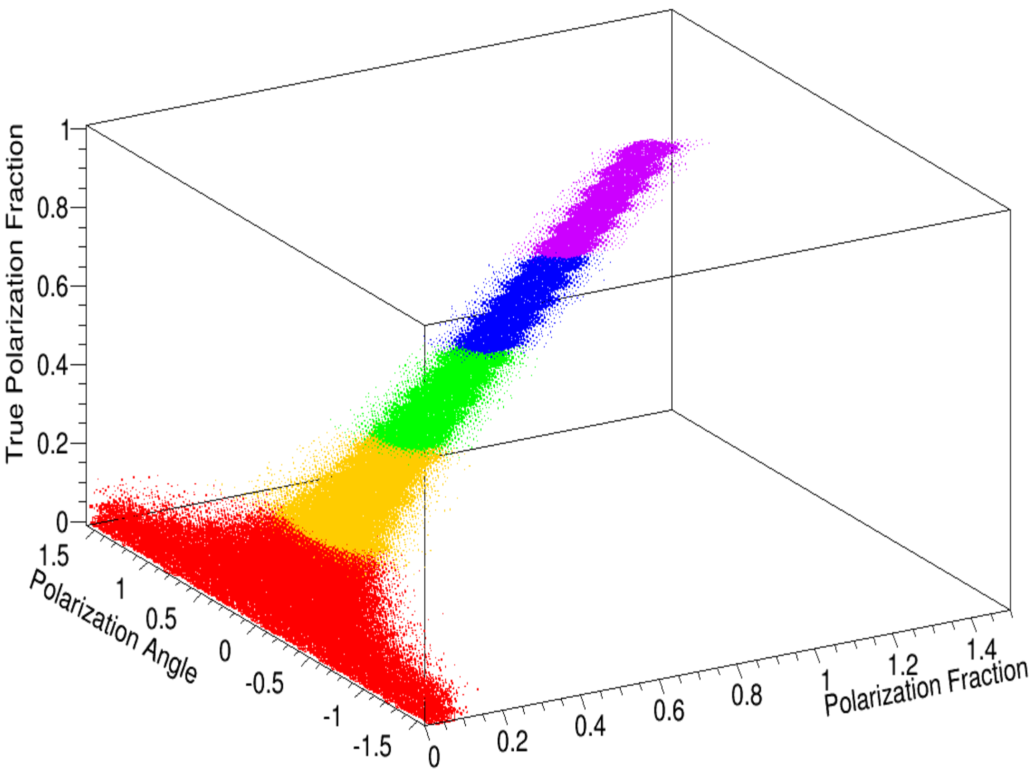
π_0=0.5, χ_0=0, μ=0.5
500 events
1000 measurements

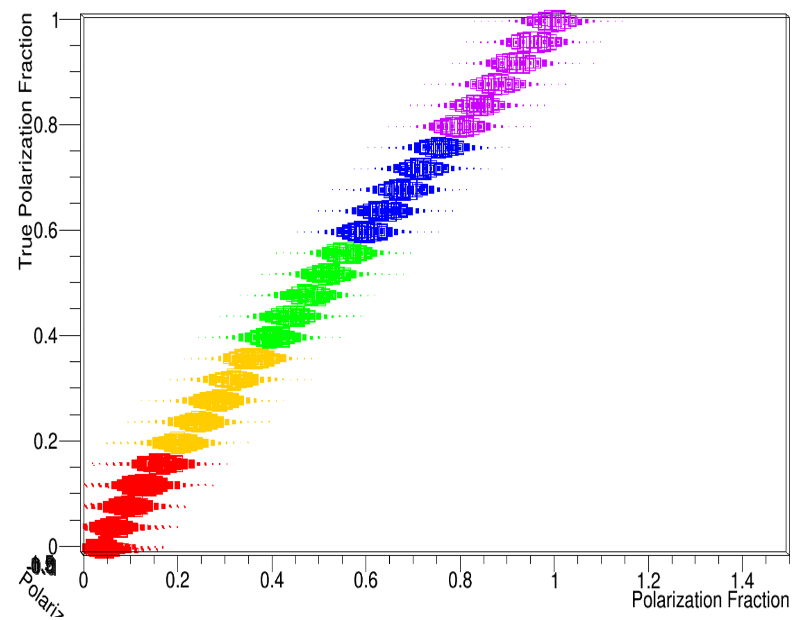
Methods

Steps 4→5: Evaluation for Many π_0 and Normalization

3- Repeat steps 1-3 (generation of a “slice” of π_0) for all π_0 ,0→1.

4- Normalize each slice (so sum of all probabilities is 1).





y-projection (down the pol angle axis)

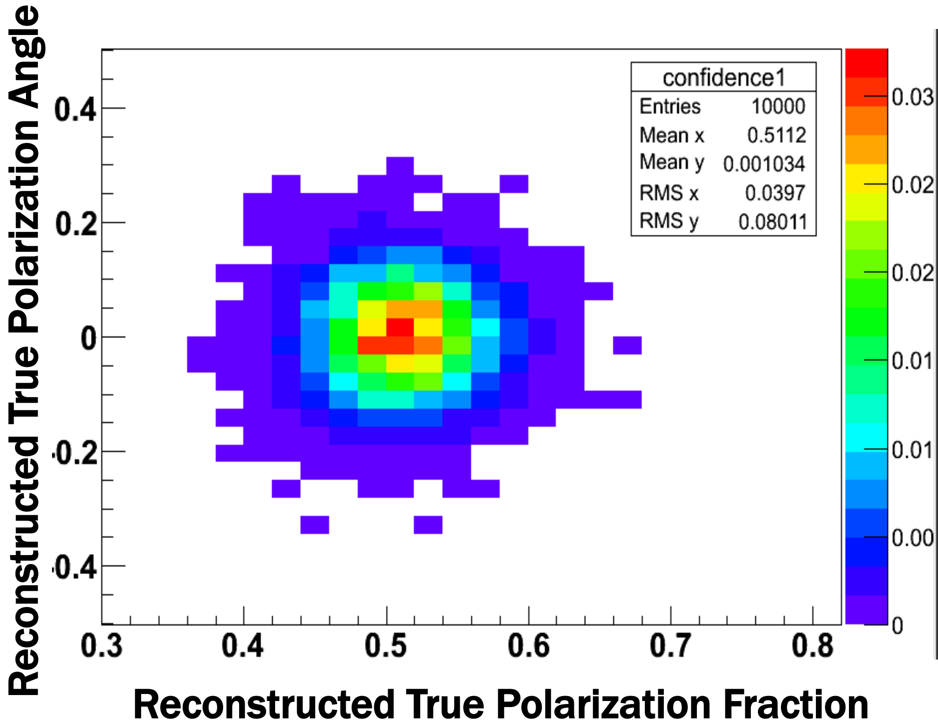
π_0=.5, χ_0=0
5000 events, 10000 measurements

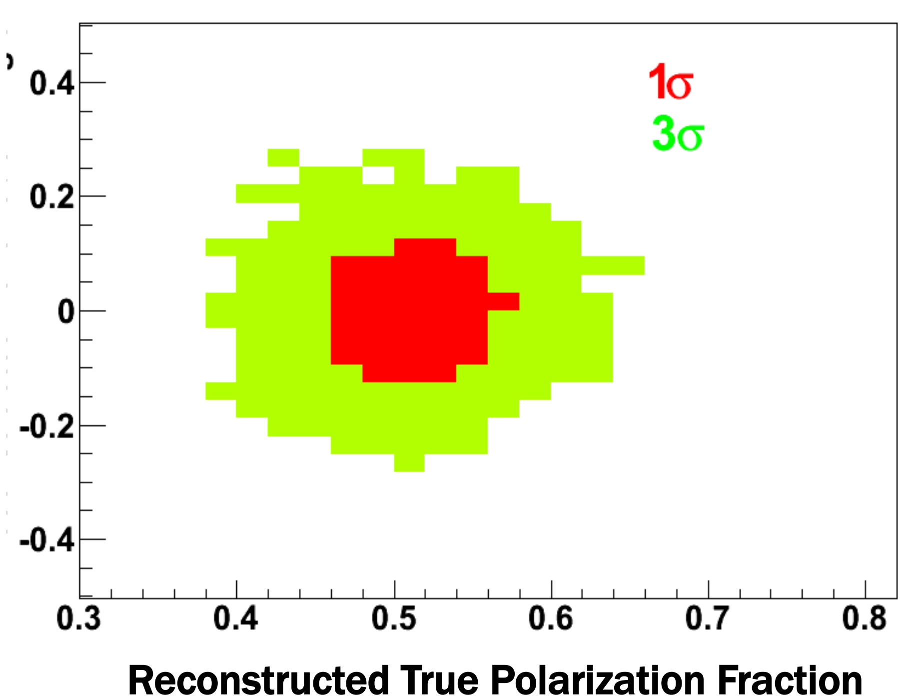
Methods

Steps 6→7: Inversion and Interval Extraction

6- Invert the 3-D distribution according to Bayes' Theorem (is a rank 4 tensor, stored in a N-dimensional sparse matrix)

7- Extract confidence interval for a (π,χ) of interest—tells you with what confidence your measured (π,χ) represent (π_0,χ_0).





Conclusion

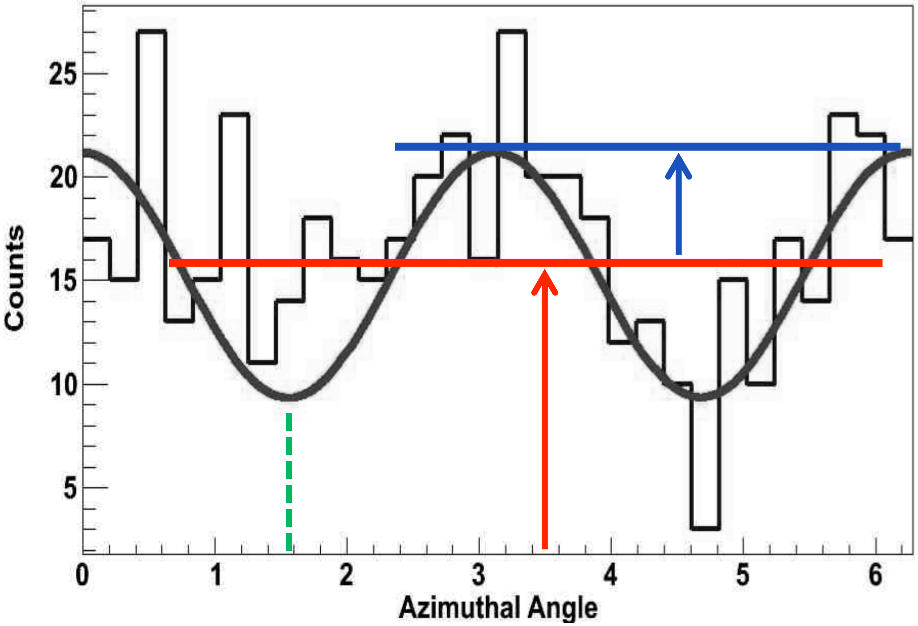
- Successfully applied Bayes theorem to generate confidence intervals and set error bars on a measured (π, χ)
- Future research interest and next steps
  - Raising the number of measurements
  - Assuming a non flat prior

Thank You!

Questions?

EXTRA SLIDES

Analysis Techniques—Fitting



Minimum= Polarization Angle

Amplitude

Mean Value

Polarization determined by fitting sine waves to distributions of azimuthal scattering angles, and extracting relevant fit parameters.

Polarization Fraction π = Amplitude / (Mean Value*μ)

Polarization Angle= Phase

Source of the sinusoidal signal in the azimuth: the Klein-Nishina

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{r_0^2}{2}\frac{k_1^2}{k_0^2}\left[\frac{k_0}{k_1} + \frac{k_1}{k_0} - 2\sin^2\theta\cos^2\chi\right]$$

Analysis Techniques—Stokes Parameters

- Utilizes the "maximum amount of available information"
  - Working directly with the discrete photons avoids information lost when they are binned in the azimuthal histogram
- Avoids pitfalls of binning and fitting
  - Measured fraction distribution systematically broaden as histogram bin number rises
  - Computed fraction and angle depend intimately on the goodness of fit of the wave—problematic for low event numbers
- Note: Should always plot the azimuth to ensure signal is *actually* sinusoidal; the Stokes cannot tell you that