GROUND BASED SEARCHES FOR GRAVITATIONAL WAVES

Rainer Weiss, MIT
Workshop on TeV Particle Astrophysics
June 27, 2006
Madison, Wisconsin
Outline

• Basic concepts
• Noise in the detectors
• Current projects
• Program for improvements in sensitivity
• Sources for ground based detectors
• Sampling of results
Direct detection of gravitational waves from astrophysical sources

Physics
» Observations of gravitation in the strong field, high velocity limit
» Determination of wave kinematics – polarization and propagation
» Tests for alternative relativistic gravitational theories

Astrophysics
» Measurement of coherent inner dynamics – stellar collapse, pulsar formation…. 
» Compact binary coalescence – neutron star/neutron star, black hole/black hole
» Neutron star equation of state
» Primeval cosmic spectrum of gravitational waves

Gravitational wave survey of the universe
test mass

light storage arm

beam splitter

power recycling mirror

laser

photodetector

test mass

light storage arm
Measurement challenge

- Needed technology development to measure:

\[ h = \frac{\Delta L}{L} < 10^{-21} \]

\[ \Delta L < 4 \times 10^{-18} \text{ meters} \]
FRINGE SENSING

wavelength $1 \times 10^{-6}$ m

$$h = \frac{x}{L} \sim \frac{\lambda}{L b \sqrt{N \tau}}$$

arm length = 4000 m

integration time

equivalent # of passes = 100

number of quanta/second at the beam splitter

300 watts at beam splitter = $10^{21}$ identical photons/sec

$$h = 6 \times 10^{-22} \quad \text{integration time} \ 10^{-2} \ \text{sec}$$
PENDULUM THERMAL NOISE

Pendulum Brownian motion

Dissipation leads to fluctuations

\[ T_c = \text{coherence or damping time} = Q \times \text{period of oscillator} \]

Exchange with surroundings:

\[ E(\text{thermal}) = \frac{kT}{T_c} t \]

Large \( T_c \) => smaller fluctuations
Quantum Noise in the Michelson Interferometer

\[ F_{\text{rad}} = (E_{\text{coh}} + E_{\text{incA}} + E_{\text{incS}})^2 \]

\[ \Delta F_{\text{rad}} = 4E_{\text{incA}}E_{\text{coh}} \]

\[ E_{\text{incA}} = E_{\text{incS}} \sim \sqrt{\hbar v\Delta B} \]

\[ E_{\text{coh}} \sim \sqrt{n\hbar v} \]

\[ E_A = E_{\text{incA}} + \frac{\Delta F_{\text{rad}} E_{\text{coh}}}{m\omega^2\lambda} \]
Interferometers

International network

Simultaneously detect signal (within msec)

detection confidence
locate the sources
decompose the polarization of gravitational waves
LIGO Observatory Facilities

**LIGO Hanford Observatory [LHO]**
26 km north of Richland, WA

2 km + 4 km interferometers in same vacuum envelope

**LIGO Livingston Observatory [LLO]**
42 km east of Baton Rouge, LA

Single 4 km interferometer
LIGO Directorate

- Jay Marx    Caltech    Director of LIGO Laboratory
- Albert Lazzarini    Caltech    Deputy Director
- Peter Saulson    Syracuse    Spokesperson LIGO Scientific Collaboration
- Stan Whitcomb    Caltech    LIGO Laboratory Scientific Leader
- David Shoemaker    MIT    MIT Group leader
LIGO Scientific Collaboration Member Institutions

University of Adelaide ACIGA
Australian National University ACIGA
Balearic Islands University
California State Domíquez Hills
Caltech CACR
Caltech LIGO
Caltech Experimental Gravitation CEGG
Caltech Theory CART
University of Cardiff GEO
Carleton College
Cornell University
Fermi National Laboratory
University of Florida @ Gainesville
Glasgow University GEO
NASA-Goddard Spaceflight Center
University of Hannover GEO
Hobart – Williams University
India-IUCAA
IAP Nizhny Novgorod
Iowa State University
Joint Institute of Laboratory Astrophysics
Salish Kootenai College

LIGO Livingston LIGOLA
LIGO Hanford LIGOWA
Loyola New Orleans
Louisiana State University
Louisiana Tech University
MIT LIGO
Max Planck (Garching) GEO
Max Planck (Potsdam) GEO
University of Michigan
Moscow State University
NAOJ - TAMA
Northwestern University
University of Oregon
Pennsylvania State University
Southeastern Louisiana University
Southern University
Stanford University
Syracuse University
University of Texas@Brownsville
Washington State University@ Pullman
University of Western Australia ACIGA
University of Wisconsin@Milwaukee
Combined

Range Trend

\[\text{In}[184]:=\]
\[\text{pltHHL = ShowLegend[}\]
\[\text{Show}\{\text{pltH1, pltH2, pltL1}, \text{PlotRange}\to \{\text{All, \{0, 16\}}\}, \text{DisplayFunction}\to \text{Identity}\},\]
\[\{\{\text{RGBColor[1, 0, 0]}, \text{"H1"}\}, \{\text{RGBColor[0, 0, 1]}, \text{"H2"}\}, \{\text{RGBColor[0, 0.6, 0]}, \text{"L1"}\}\},\]
\[\text{LegendPosition}\to \{0.6, 0.45\}, \text{LegendSize}\to \{0.3, 0.2\}, \text{LegendShadow}\to \{0.02, -0.02\}\};\]
Duty Cycle Trend

```
In[189]:= histHHL = ShowLegend[Show[{pltH1avrg, pltH2avrg, pltL1avrg}, DisplayFunction -> Identity], (({RGBColor[1, 0, 0], "H1"}, {RGBColor[0, 0, 1], "H2"}, {RGBColor[0, 0.6, 0], "L1"}), LegendPosition -> {-0.9, -0.45}, LegendSize -> {0.4, 0.3}, LegendShadow -> {0.02, -0.02}]);
```
L1 in S5: Where Has The Time Gone?
Nov23 - Jun19 (Seg110-2230)

Uptime: 56.5%

- 9.6% - Construction & Logging
- 4.2% - Earthquakes
- 1.8% - Unknown
- 2.6% - Microseism
- 3.2% - Wind & Storms
- 3% - Up/Down Scripts
- 1.3% - Software
- 9.3% - Maintenance & Commissioning
- 2.9% - Hardware
- 0.4% - Injections & Calibration
Program of improvements

- **Major steps between initial and advanced LIGO**
  - Increase laser input power 10 to 180 watts in stages
  - Incorporation of an output mode cleaner
  - Output optics and electro-optics chain in vacuum
  - DC (carrier offset) “modulation” technique
  - Reduction in thermal noise
    - Steel wire to fused quartz ribbon suspension elements
    - Lower mechanical dissipation optical coatings
    - Larger test masses: 10 kg to 40 kg
  - Improved seismic isolation – extend sensitivity to 15Hz
  - Tunable dual recycling interferometer configuration
  - Quantum limited operation over significant band
Strain sensitivity initial, enhanced and advanced LIGO

- INIT LIGO SRD
- advligo low freq tuning
- advligo narrow band tuning
- enhanced initial LIGO NS/NS tuning
- gravg grad coating thermal suspension thermal advligo narrow band tuning
Classes of sources

• Compact binary inspiral: template search
  – BH/BH
  – NS/NS and BH/NS

• Low duty cycle transients: wavelets, T/f clusters
  – Supernova
  – BH normal modes
  – Unknown types of sources

• Periodic CW sources
  – Pulsars
  – Low mass x-ray binaries (quasi periodic)

• Stochastic background
  – Foreground sources: gravitational wave radiometry
  – Cosmological isotropic background
Binary Coalescence Sources & Science:
Binary Neutron Stars: LIGO Range

Image: R. Powell
Binary Coalescence Sources & Science:
Binary Neutron Stars: AdLIGO Range
Search for binary systems

- Search for double or triple coincident “triggers”
- Estimate false alarm probability of resulting candidates: detection?
- Compare with expected efficiency of detection and surveyed galaxies: upper limit

B. Abbott et al. (LIGO Scientific Collaboration):
- S2: Search for gravitational waves from primordial black hole binary coalescences in the galactic halo, Phys. Rev. D 72, 082002 (2005)
- S2: Search for gravitational waves from galactic and extra-galactic binary neutron stars, Phys. Rev. D 72, 082001 (2005)
- S2: Search for gravitational waves from binary black hole inspirals in LIGO data, Phys. Rev. D 73, 082001 (2006)
- S2: Joint Search for Gravitational Waves from Inspiralling Neutron Star Binaries in LIGO and TAMA300 data (LIGO, TAMA collaborations), PRD, in press
- S3: finished searched for BNS, BBH, PBBH: no detection
- S4, S5: searches in progress.
No GWBs detected through S4. So, set limit on GWB rate vs. signal strength:

\[ R(h_{rss}) = \frac{\eta}{\varepsilon(h_{rss}) \times T} \]

- \( \eta \) = upper limit on event number
- \( T \) = observation time
- \( \varepsilon(h_{rss}) \) = efficiency vs strength

Progress:

- Lower rate limits from longer observation times
- Lower amplitude limits from lower detector noise

Latest (unpublished) results in Session W11:
- Shawhan – Science Run 4
- Yakushin – Science Run 5
h_0 Results

- Spin-down upper limit calculated with intrinsic spin-down value if available i.e. corrected for Shklovskii transverse velocity effect
- Closest to spin-down upper limit
  - Crab pulsar \( \sim 2.1 \) times greater than spin-down (\( f_{gw} = 59.6 \) Hz, dist = 2.0 kpc)
  - \( h_0 = 3.0 \times 10^{-24} \), \( \varepsilon = 1.6 \times 10^{-3} \)
  - Assumes \( I = 10^{38} \) kgm\(^2\)

- Sensitivity curves use:

\[
S(f) = \left( \frac{T_{obs \ H1} h_{H1}(f) + T_{obs \ H2} h_{H2}(f) + T_{obs \ L1} h_{L1}(f)}{S_h(f)_{H1} + S_h(f)_{H2} + S_h(f)_{L1}} \right)^{-1}
\]

\[h_0^{95\%} = 10.8 \sqrt{S(f)}.\]
### S5 Results – 95% upper limits

<table>
<thead>
<tr>
<th>$h_0$</th>
<th>Pulsars</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \times 10^{-25} &lt; h_0 &lt; 5 \times 10^{-25}$</td>
<td>44</td>
</tr>
<tr>
<td>$5 \times 10^{-25} &lt; h_0 &lt; 1 \times 10^{-24}$</td>
<td>24</td>
</tr>
<tr>
<td>$h_0 &gt; 1 \times 10^{-24}$</td>
<td>5</td>
</tr>
</tbody>
</table>

**Lowest $h_0$ upper limit:**
PSR J1603-7202 ($f_{gw} = 134.8$ Hz, $r = 1.6$ kpc) $h_0 = 1.6 \times 10^{-25}$

**Lowest ellipticity upper limit:**
PSR J2124-3358 ($f_{gw} = 405.6$ Hz, $r = 0.25$ kpc) $\varepsilon = 4.0 \times 10^{-7}$

### Ellipticity

<table>
<thead>
<tr>
<th>Ellipticity</th>
<th>Pulsars</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon &lt; 1 \times 10^{-6}$</td>
<td>6</td>
</tr>
<tr>
<td>$1 \times 10^{-6} &lt; \varepsilon &lt; 5 \times 10^{-6}$</td>
<td>28</td>
</tr>
<tr>
<td>$5 \times 10^{-6} &lt; \varepsilon &lt; 1 \times 10^{-5}$</td>
<td>13</td>
</tr>
<tr>
<td>$\varepsilon &gt; 1 \times 10^{-5}$</td>
<td>26</td>
</tr>
</tbody>
</table>

All values assume $I = 10^{38}$ kgm$^2$ and no error on distance

$$ \varepsilon = 0.237 \frac{h_0}{10^{-24}} \frac{r}{1 \text{ kpc}} \frac{1 \text{ Hz}^2}{v^2} \frac{10^{38} \text{ kgm}^2}{I_{zz}} $$
Predictions and Limits

\[ \Omega_{GW}(f) = \frac{1}{\rho_c} \frac{d\rho_{GW}(f)}{d \ln f} \]

\[ H_0 = 72 \text{ km/s/Mpc} \]

LIGO S1: \( \Omega_0 < 44 \)
PRD 69 122004 (2004)

LIGO S3: \( \Omega_0 < 8.4 \times 10^{-4} \)
PRL 95 221101 (2005)

LIGO S4: \( \Omega_0 < 6.5 \times 10^{-5} \)
(new)

Initial LIGO, 1 yr data
Expected Sensitivity
~ 4 \times 10^{-6}

Advanced LIGO, 1 yr data
Expected Sensitivity
~ 1 \times 10^{-9}

Cosmic strings
Pulsar Timing
BB Nucleosynthesis
Pre-big bang model
CMB
Inflation
Slow-roll
Cyclic model
EW or SUSY
Phase transition

LIGO S4:

\[ \Omega_0 < 6.5 \times 10^{-5} \]
(new)

Initial LIGO, 1 yr data
Expected Sensitivity
~ 4 \times 10^{-6}

Advanced LIGO, 1 yr data
Expected Sensitivity
~ 1 \times 10^{-9}
Similarly, FIG. 5 shows a histogram of the bias-corrected SNR = \frac{S}{N} for the constant strain power case. Structure in the spherical harmonics power spectrum goes up to \( l = 19 \), thus \( N_{\text{eff}} \) was estimated to be \( N_{\text{eff}} \approx (l + 1)^2 = 400 \). Alternatively the FWHM area of a strong injection covers about 100 deg\(^2\) which also leads to \( N_{\text{eff}} \approx 400 \). The dash-dotted red lines in the histogram (FIG. 5) correspond to the expected 1 − \( \sigma \) deviations from the ideal Gaussian for \( N_{\text{eff}} = 400 \). The histogram is thus consistent with (correlated) Gaussian noise, indicating that there is no signal present. The SNR distribution also passes a Kolmogorov-Smirnov test for \( N_{\text{eff}} = 400 \) (\( \alpha = 0.1 \)).

FIG. 5: S4 Result: Histogram of the bias corrected signal-to-noise ratio (SNR) for \( H(f) = 1 \) Hz\(^{-1}\). The green curve is a maximum likelihood Gaussian fit to the data. The red solid line is an ideal Gaussian, the two dash-dotted red lines are the 1 − \( \sigma \) bands around the ideal Gaussian for \( N_{\text{eff}} = 400 \).

Again we calculated a Bayesian 90% upper limit for each sky direction, including the marginalization over the calibration uncertainty. The prior was assumed to be flat between 0 and \( \infty \). The resulting upper limit map is shown in FIG. 6. The upper limits on the strain power spectrum \( H(f) \) vary between \( 8.5 \times 10^{-49} \) Hz\(^{-1}\) and \( 6.1 \times 10^{-48} \) Hz\(^{-1}\) depending on the position in the sky.

FIG. 6: S4 Result: Map of the 90% confidence level Bayesian upper limit for \( H(f) = 1 \) Hz\(^{-1}\). The upper limit varies between \( 8.5 \times 10^{-49} \) Hz\(^{-1}\) and \( 6.1 \times 10^{-48} \) Hz\(^{-1}\) depending on the position in the sky.

3. Interpretation

The maps presented in FIG. 4 and 6 represent the first directional upper limits on a stochastic gravitational wave background ever obtained. They are consistent with no gravitational wave background being present.

One can interpret this result in terms of potential sources. As an example we look at the gravitational luminosity of all low-mass X-ray binaries (LMXBs) within the Virgo galaxy cluster. They have an integrated X-ray luminosity of about \( 1 \times 10^{-9} \) erg/sec/cm\(^2\) (3000 galaxies at 15 Mpc, \( 10^{-9} \) erg/sec/galaxy from LMXBs). For simplicity we assume that they produce a flat strain power spectrum \( H(f) \) over a bandwidth \( \Delta f \). Then the strength of this strain power spectrum is about

\[
H(f) = \frac{2G}{\pi c^3} \frac{1}{f_{\text{Repl}} f_{\text{center}} \Delta f} F_X
\approx 10^{-55} \text{ Hz}^{-1} \left( \frac{100 \text{ Hz}}{f_{\text{center}}} \right) \left( \frac{100 \text{ Hz}}{\Delta f} \right)
\]

which is out of reach. Here \( f_{\text{Repl}} \approx 2 \text{ kHz} \) is final orbital frequency of accreting matter and \( f_{\text{center}} \) is the typical frequency of the \( \Delta f \) wide band of interest.

B. Limits on isotropic background

It is possible to recover the point estimate and standard deviation for an isotropic background as an integral over the map (see [3]). From that the 90% Bayesian upper limit can be calculated, which is additionally marginalized over the calibration uncertainty. In the \( \beta = -3 \) case the 90% upper limit we can set on \( h^2 \Omega_{gw}(f) \) is \( 6.25 \times 10^{-5} \). Table I summarizes the results for all choices of \( \beta \).

1. Interpretation

The limit on an isotropic stochastic background of gravitational waves that can be set with the S4 data is roughly one order of magnitude lower than the published LIGO S3 limit [1]. In [? ] LIGO already published an isotropic upper limit using S4 data. While the reconstruction of the isotropic result done here is in principle identical to the analysis presented in [? ], the actual data cuts were sufficiently different that the result varies slightly. In the future initial LIGO has the
TYPE II SUPERNOVA

• Candidates for neutrino and gravitational wave detection (maybe only in our galaxy)

• SN futures
  – Early 1980’s  \(10^{-2}\) conversion \(E_{gw}/E_{total}\)
  – Late 1980’s  \(10^{-5}\)
  – Mid 1990’s  \(10^{-9}\)
  – Early 2000’s  \(10^{-7}\)
  – Now  \(10^{-4}\)

• Interesting challenges from SN
Signal from a stellar collapse to a neutron core

Saenz, R. A., Shapiro, S.
Fig. 21.—Same as Fig. 10, but for the s11A500 sequence and for \( \beta \)'s from 0.1% to 0.5%.
Fig. 22.—Same as Fig. 20, but with various full $h_{\text{tot}}$ spectra (using eq. 25) superposed. This plot makes clear the large width of actual spectra and the deviation from even quasi-periodic behavior of rotating collapse wave signatures.
gravitational-wave strain. We employ the Shen equation of state \( \Sigma \).

We explore three models in this study. Model s11WW is the 11-M_\odot (Zero-Age Main Sequence [ZAMS]) supernova model of Woosley & Weaver \( ^{17} \) without rotation. Model s25WW is nonrotating as well, but is the 25-M_\odot progenitor from the same study. Model m15b6 corresponds to the 15-M_\odot progenitor model of Heger et al. \( ^{2} \) which was evolved with a 1D prescription for rotation and magnetic-field-driven angular momentum redistribution. We map this model onto our 2D grid under the assumption of constant rotation on cylinders. It has a precollapse ratio of rotational kinetic energy to gravitational potential energy, \( \beta = T/|W| \), of \( \sim 1 \times 10^{-3} \% \). This value is one to two orders of magnitude smaller than in previous models (e.g., \( ^{1, 7, 18} \)), but yields a PNS consistent with neutron star birth spin estimates \( ^{4} \).

We extract gravitational waves from the mass motions via the quadrupole formula as described in \( ^{1, 14, 21} \). In addition, we estimate the gravitational-wave emission by anisotropic neutrino radiation with the formalism introduced by \( ^{21} \) and concretized in \( ^{1, 20} \).

**Results.** Figure 1 depicts the quadrupole gravitational wave strain \( h_+ \) as emitted by mass motions scaled to a source distance of 10 kiloparsecs (kpc). In the top panel, we superpose the waveforms of models s11WW and m15b6. Despite the presence of some rotation in the latter and its greater ZAMS mass, the two models have very similar precollapse stellar structures \( ^{2, 11, 17} \). This is reflected in the very similar shapes of their waveforms. Even though s11WW is not rotating, a bounce burst strain of \( \sim 1.3 \times 10^{-21} \) (\( @10 \) kpc) is present in our numerical model. The first to two milliseconds of this burst are the imprint of the transition in grid geometry from the outer polar to the inner Cartesian grids which generates a time-varying quadrupole moment at core bounce. It also induces initial perturbations for vortical motion in the Ledoux-unstable regions behind the expanding shock that sets in almost immediately and with a perhaps too fast initial growth rate after core bounce. The amount of rotational energy in m15b6’s core (at bounce, \( \beta \lesssim 0.02\% \) and at the end, \( \beta \sim 0.08\% \)) is too small to have a large influence on the core dynamics and, thus, on the waveform, except to slightly stabilize the aspherical fluid motion at and shortly after bounce. In both models, until about \( \sim 250 \) ms after bounce the physical waveform is dominated by convective motions in the PNS and in the post-shock region. As the SASI \( ^{9, 10, 11, 12} \) becomes vigorous and leads to global deformation of the standing shock (see, e.g., \( ^{2} \)) the wave emission from the post-shock flow increases.

As described in Burrows et al. \( ^{8} \) the fundamental core g-mode (\( \ell=1, f \sim 300 \) Hz in s11WW and m15b6) is excited by turbulence and accretion. It grows strong around \( \sim 400 \) ms after bounce and starts transferring energy to the harmonic at 2f through nonlinear effects. This is reflected in the rise of s11WW’s gravitational-wave strain around that time. \( h_+ \) reaches a local maximum, then quickly decays to about one-third that amplitude, only to pick up again after some tens of milliseconds, rising to even higher amplitudes (a maximum of \( \sim 7 \times 10^{-22} \) [\( @10 \) kpc]), followed by a quasi-exponential decay with a \( \sim 100 \) ms e-folding time. We attribute the gravitational-wave emission in these two ‘humps’ to the quadrupole spatial component of the 2f harmonic of the \( \ell=1 \) core g-mode. s11WW’s gravitational-wave energy spectrum exhibits prominent emission in a band around \( \sim 650 \) Hz. A frequency analysis shows that the harmonic appears first at a frequency of \( \sim 590 \) Hz, which increases over 200 ms to a maximum of about 680 Hz, and then continuously decreases to \( \sim 500 \) Hz at the end of the simulation. In this way, the gravitational-wave emitting component exactly mirrors the behavior of the \( \ell=1 \) g-mode which goes through the same phases \( ^{8, 22} \). This behavior is qual-

![A. Burrows et al. 2006](image-url)


tance to the source. This is a particularly useful measure, since it incorporates the amount of energy radiated in a spectral interval \( df \) around \( f \). In addition, we show the optimal \( h_{\text{rms}} = \sqrt{S(f)} \), \( S(f) \) being the spectral strain sensitivity) of both LIGO I and Advanced LIGO [26]. All \( h_{\text{char}} \) spectra peak strongly at the frequencies identified with the quadrupole components of the core oscillations (Fig. 4) between 600 and 1000 Hz, likely to be higher when general relativity is included), corroborating the narrow-band nature of the emission process. Given our results, we conclude that, if the core oscillations observed in our simulations are generically excited in core-collapse supernovae, even nonrotating supernovae of small to intermediate progenitor mass should be observable by LIGO throughout the Milky Way and beyond. Massive progenitors could be detectable out to \( \sim 100 \) times greater distances and their prolonged and extreme energetics core-oscillation wave signature might be the generic precursor of stellar-mass black-hole formation.

We point out that the work presented here is based on simulations in 2D Newtonian gravity and only quadrupole wave emission has been considered. General relativity is likely to increase the frequency of the PNS eigenmodes, but is unlikely to lead to qualitative differences. Fast rotation might lead to the partial stabilization of the post-shock convection, might affect the growth of core oscillations, and will likely lead to non-axisymmetric rotational instabilities for \( \beta \gtrsim 8\% \). In 3D, the temporal and spatial mode and SASI structures may change.

We acknowledge help from and discussions with C. Meakin, J. Murphy, H. Dimmelmeier, E. Müller, J. Pons, N. Stergioulas, L. Rezzolla, and E. Seidel. We acknowledge support from the Scientific Discovery through Advanced Computing (SciDAC) program of the DOE, grant number DE-FC02-01ER41184, and from the NSF, grant number AST-0504947. E. L. acknowledges support from the Israel Science Foundation under grant 805/04. This research used resources of the National Energy Research Scientific Computing Center, which is supported by the Office of Science of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.


FIG. 4: Characteristic strain spectra contrasted with initial and advanced LIGO (optimal) rms noise curves.
the overall GW event rate may be worse by about a factor 2.

Figure 1 shows, along with the noise curves, the estimated signal strengths $\tilde{h}(f)$ for various sources. These signal strengths are defined in such a way that the ratio $\tilde{h}_s(f)/\tilde{h}(f)$ is equal to the ratio of signal $S$ to noise threshold $T$, rms averaged over source directions and orientations, $\tilde{h}_s(f)/\tilde{h}(f) = \langle S^2/T^2 \rangle^{1/2}$, with the threshold being that at which the false alarm probability is one per cent when using the best currently known, practical data analysis