**RC summary**

- **charge:**
  
  \[ \varepsilon = R \frac{dq}{dt} + \frac{q}{C} \]

  - \( q = C \varepsilon (1 - e^{-t/RC}) \)
  - The *time constant*, \( \tau = RC \)

  \[ I(t) = \frac{\varepsilon}{R} e^{-t/RC} \]

- **discharge:**
  
  \( q/C = RI \quad I = -dq/dt \)

  - \( q = Qe^{-t/RC} \)

  \[ I(t) = -\frac{dq}{dt} = -\frac{Q}{RC} e^{-t/RC} \]
Work to charge a capacitor

- The final charge is $Q = C\varepsilon$
- $W = Q\varepsilon = C\varepsilon^2$
- $1/2$ of this work is energy stored in the capacitor $U = 1/2C\varepsilon^2$
- energy dissipated in $R$

$$W_R = \int_0^\infty R\frac{\varepsilon^2}{R^2}e^{-2t/RC} \, dt = \frac{\varepsilon^2}{R} \frac{RC}{2} = \frac{1}{2} C\varepsilon^2$$
Membranes as capacitors

- Lipid bilayers of cell membranes are like plates of a capacitor made of polar lipid heads separated by a dielectric layer of hydrocarbon tails.
- Due to the ion distribution inside and outside the cell, there is a potential difference called the resting potential.
- Ion channels allow current flow across the membrane.
Membrane Electrical Model

- **specific resistance of membranes:** \( R = \frac{\rho L}{A} \Rightarrow R A = \frac{\rho L}{10^{-10}} \, \Omega \, \text{cm}^2 \)
- **capacitance/area** \( \frac{C}{A} = 1 \, \mu\text{F}/\text{cm}^2 \)
- The simplest model for a biological membrane is an RC circuit
- When S is closed (= membrane channel opens) \( \Rightarrow \) discharge of C
- RC time constants range from \( 10 \, \mu\text{s} \) to \( 1 \, \text{s} = (RA)(C/A) \)

A) Basic RC circuit for a membrane superimposed to the bilayer image and a membrane with ion channels

B) Cell membrane with ion channels
This Lecture (Ch 26 T&M):
- Magnetic Fields and Magnets
- Motion of charges in Magnetic Fields
- Force on a wire with current
- Torques on Current loops
Magnetic Fields in ordinary life

Aurora Borealis
North magnetic pole is about halfway around the Earth ($\pi R_E$) from the North geographic pole

- N geographic pole almost at magnetic S pole
- S geographic pole almost at magnetic N pole

William Gilbert (1600)
Magnets

- 13th century BC: Chinese already used a compass with a magnetic needle (Arabic or Indian origin?)
- 800 BC: Greeks discovered magnetite (Fe₃O₄)

- **Like poles repel each other**
  - N-N or S-S
- **Unlike poles attract each other**
  - N-S
Sources of magnetic field

- Magnetic Fields are created by moving electric charge!
- **1819** Hans Christian Oersted
  - Discovered the relationship between electricity and magnetism
  - An electric current in a wire deflected a nearby compass needle
- **1820’s**
  - Faraday and Henry
    - Further connections between electricity and magnetism
    - A changing magnetic field creates an electric field
  - Maxwell
    - A changing electric field produces a magnetic field
Magnetic Fields

- A vector quantity ($\mathbf{B}$)
- Compass needle traces $\mathbf{B}$ field lines and points towards N
- The lines outside the magnet point from N to S (do not stop because there is no monopole)

Iron filings show the pattern of the electric field lines

- B-Field lines as E-lines
  - Density gives strength
  - Arrow gives direction
    - Leave $+$, North
    - Enter $-$, South
- Differences
  - Start/Stop on electric charge
  - No Magnetic Charge $\Rightarrow$ lines are continuous!
Let’s Break A Magnet!

- Magnetic poles are always found in pairs!

A monopole has never been observed (but...!)!

Only in the region between the 2 charges E-lines differ from B ones

\[ \oint \mathbf{B} \cdot d\mathbf{A} = 0 \]
B-field representation

(a) B out of page:

(b) B into page:
\[ \mathbf{F}_B = q \mathbf{v} \times \mathbf{B} \]

- \(\mathbf{v}\) is the velocity of the moving charge \(q\)

- **SI unit of magnetic field**: tesla (T)
  \[ T = \frac{\text{Wb}}{m^2} = \frac{N}{C \cdot (m/s)} = \frac{N}{A \cdot m} \]

- **CGS unit**: gauss (G):
  \[ 1 \text{ T} = 10^4 \text{ G} \text{ (Earth surface 0.5 G)} \]

- The magnetic force does no work when a particle is displaced (force perpendicular to displacement)

- The field can alter the *direction* of the velocity, but not the speed or the kinetic energy
Lorentz Force

\[ \vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \]
Effect of uniform magnetic field on a charged particle in motion

- If particle is moving: force exerted **perpendicular** to both field and velocity
Trajectory in Constant B orthogonal to plane of motion

Path is a circle. F/m is the centripetal acceleration of the particle in circular orbit.

\[ F = ma \]

\[ qvB = m \frac{v^2}{R} \Rightarrow R = \frac{mv}{qB} \]

\[ \omega = \frac{qB}{R} \]

The cyclotron angular frequency does not depend on v