Waves

- Oscillations (e.g. pendulum, spring-mass system)
- Waves - space & time dependent oscillations
- Light waves - interference & diffraction

Lead us to Quantum Theory: particles are waves!

In fact, the uncertainty principle can be understood in terms of diffraction.

\[ \Delta p \Delta x \geq \frac{\hbar}{2} \]

Simpest example of oscillations

![Diagram of a spring-mass system with displacement of spring labeled Hoëne's law]
\[ ma = -kx \]
\[ m \frac{d^2x}{dt^2} = -kx \]
\[ \frac{d^2x}{dt^2} = - \frac{k}{m} x \equiv -\omega^2 x \quad \omega = \sqrt{\frac{k}{m}} \text{ angular freq.} \]

To solve this equation, let's recall that sine and cosine have the property that their 2nd derivatives are proportional to themselves:
\[ \frac{d^2}{dx^2} \sin x = -\sin x \]
\[ \frac{d^2}{dx^2} \cos x = -\cos x \]

**Guess:** \[ x(t) = A \cos (\omega t + \phi) \]

\[ A = \text{amplitude } (\text{arbitrary set by initial conditions}) \]
\[ \phi = \text{phase} \]

Note that by including \( \phi \), we cover both the cases of sine and cosine.

**Check:**
\[ \frac{dx}{dt} = -A\omega \sin (\omega t + \phi) \]
\[ \frac{d^2x}{dt^2} = -A\omega^2 \cos (\omega t + \phi) = -\omega^2 A \]
\[ \omega T = 2\pi \quad \text{cosine repeats} \]

\[ \omega = \frac{2\pi}{T} \]

Frequency \[ f = \frac{1}{T} = \# \text{ of cycles per unit time} \]

\[ = \frac{\omega}{2\pi} \]

Initial conditions:

say \[ x(t=0) = x_0 \]
\[ \dot{x}(t=0) = 0 \]

\[ \Rightarrow A \cos \delta = x_0 \]

\[ -A \omega \sin \delta = 0 \]

\[ \Rightarrow \delta = 0 \quad (\text{actually integer multiple of } \pi) \]
\[ A = x_0 \]
Example: Circular motion

\[ \theta = \omega t \quad \text{constant speed} \]
\[ x = r \cos \theta = r \cos \omega t \]
\[ y = r \sin \theta = r \sin \omega t \]

Solution is exactly the type \( x = A \cos(\omega t + \delta) \) discussed before.

Energy of an oscillatory system

\[ E = K + U \]
\[ = \frac{1}{2} mv^2 + \frac{1}{2} kx^2 \]
\[ = \frac{1}{2} m \left( -\omega A \sin(\omega t + \delta) \right)^2 + \frac{1}{2} \left( ma^2 \right) \left[ A \cos(\omega t + \delta) \right] \]
\[ = \frac{1}{2} mw^2 A^2 \left[ \sin^2(\omega t + \delta) + \cos^2(\omega t + \delta) \right] \]

\[ = \frac{1}{2} mw^2 A^2 \uparrow \text{amplitude}^2 \]

\[ \downarrow \text{fixed} \]
Simple Harmonic Oscillation (SHO)

Very common in physics.

\[ F = -\frac{du}{dx} \]

\[ m \frac{d^2x}{dt^2} = -\frac{du}{dx} \]

Expand around minimum \( x_0 \)

\[ U(x) = U(x_0) + U'(x_0)(x-x_0) + \frac{1}{2} U''(x_0)(x-x_0)^2 + \cdots \]

\[ 0 \text{ if } x_0 \text{ is a minimum} \]

\[ = U(x_0) + \frac{U''(x_0)}{2}(x-x_0)^2 \]

\[ F = -\frac{du}{dx} = -U''(x_0)(x-x_0) \] \( \text{just like Hooke's law} \)
Small oscillations about $x_0$

\[ k = \frac{d^2U}{dx^2} \bigg|_{x=x_0} \]

\[ \omega = \sqrt{\frac{k}{m}} \]

To increase $\omega$, we can increase the curvature of $U$

**Example** vertical spring

\[ U = \frac{1}{2} k z^2 + mg \]

**Equilibrium** \( \frac{du}{dz} = 0 = k z + mg \)

\[ \Rightarrow z = -\frac{mg}{k} \]

\[ U \]

No gravity

\[ z = -\frac{mg}{k} \]

w/ Gravity