Problems selected from Tipler & Mosca on Electrostatics

Ch 21 Discrete Charge Distribution

***23** ••

Picture the Problem We can use Coulomb's law to express the charge on the rod in terms of the force exerted on it by the soda can and its distance from the can. We can apply Newton's 2nd law in rotational form to the can to relate its acceleration to the electric force exerted on it by the rod. Combining these equations will yield an expression for Q as a function of the mass of the can, its distance from the rod, and its acceleration.

Use Coulomb's law to relate the force on the rod to its charge Q and distance r from the soda can:

$$F = \frac{kQ^2}{r^2}$$

Solve for Q to obtain:

$$Q = \sqrt{\frac{r^2 F}{k}} \qquad (1)$$

Apply $\sum \tau_{\text{center of mass}} = I\alpha$ to the can:

$$FR = I\alpha$$

Because the can rolls without slipping, we know that its linear acceleration a and angular acceleration α are related according to:

$$\alpha = \frac{a}{R}$$

where R is the radius of the soda can.

Because the empty can is a hollow cylinder:

$$I = MR^2$$

where M is the mass of the can.

Substitute for I and α and solve for F to obtain:

$$F = \frac{MR^2 a}{R^2} = Ma$$

Substitute for F in equation (1):

$$Q = \sqrt{\frac{r^2 Ma}{k}}$$

Substitute numerical values and evaluate Q:

$$Q = \sqrt{\frac{(0.1\,\text{m})^2(0.018\,\text{kg})(1\,\text{m/s}^2)}{8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2}} = \boxed{141\,\text{nC}}$$

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Picture the Problem Let the numeral 1 refer to the charge in the 1st quadrant and the numeral 2 to the charge in the 4th quadrant. We can use Coulomb's law for the electric field due to a point charge and the superposition of forces to express the field at the origin and use this equation to solve for Q.

Express the electric field at the origin due to the point charges Q:

$$\vec{E}(0,0) = \vec{E}_1 + \vec{E}_2 = \frac{kQ}{r_{1,0}^2}\hat{r}_{1,0} + \frac{kQ}{r_{2,0}^2}\hat{r}_{2,0}$$

$$= \frac{kQ}{r^3}\left[(-4\,\text{m})\hat{i} + (-2\,\text{m})\hat{j}\right] + \frac{kQ}{r^3}\left[(-4\,\text{m})\hat{i} + (2\,\text{m})\hat{j}\right] = -\frac{(8\,\text{m})kQ}{r^3}\hat{i}$$

$$= E_x\hat{i}$$

where r is the distance from each charge to the origin and $E_x = -\frac{(8\,\text{m})kQ}{r^3}$.

Express r in terms of the coordinates (x, y) of the point charges:

$$r = \sqrt{x^2 + y^2}$$

Substitute to obtain:

$$E_x = -\frac{(8\,\text{m})kQ}{\left(x^2 + y^2\right)^{3/2}}$$

Solve for Q to obtain:

$$Q = \frac{E_x\left(x^2 + y^2\right)^{3/2}}{k(8\,\text{m})}$$

Substitute numerical values and evaluate Q:

$$Q = -\frac{(4\,\text{kN/C})\left[(4\,\text{m})^2 + (2\,\text{m})^2\right]^{3/2}}{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)(8\,\text{m})}$$

$$= \boxed{-4.97\,\mu\text{C}}$$

55 ••

Picture the Problem We can use constant-acceleration equations to express the x and y coordinates of the electron in terms of the parameter t and Newton's 2nd law to express the constant acceleration in terms of the electric field. Eliminating the parameter will yield an equation for y as a function of x, q, and m. We can decide whether the electron will strike the upper plate by finding the maximum value of its y coordinate. Should we find that it does not strike the upper plate, we can determine where it strikes the lower plate by setting $y(x) = 0$.

Express the x and y coordinates of the electron as functions of time:

$$x = (v_0 \cos\theta)t$$

and

$$y = (v_0 \sin\theta)t - \tfrac{1}{2} a_y t^2$$

Apply Newton's 2nd law to relate the acceleration of the electron to the net force acting on it:

$$a_y = \frac{F_{\text{net},y}}{m_e} = \frac{eE_y}{m_e}$$

Substitute in the y-coordinate equation to obtain:

$$y = (v_0 \sin\theta)t - \frac{eE_y}{2m_e} t^2$$

Eliminate the parameter t between the two equations to obtain:

$$y(x) = (\tan\theta)x - \frac{eE_y}{2m_e v_0^2 \cos^2\theta} x^2 \quad (1)$$

To find y_{max}, set $dy/dx = 0$ for extrema:

$$\frac{dy}{dx} = \tan\theta - \frac{eE_y}{m_e v_0^2 \cos^2\theta} x' = 0 \text{ for extrema}$$

Solve for x' to obtain:

$$x' = \frac{m_e v_0^2 \sin 2\theta}{2eE_y} \quad \text{(See remark below.)}$$

Substitute x' in $y(x)$ and simplify to obtain y_{max}:

$$y_{\text{max}} = \frac{m_e v_0^2 \sin^2\theta}{2eE_y}$$

Substitute numerical values and evaluate y_{max}:

$$y_{\text{max}} = \frac{(9.11\times10^{-31}\text{ kg})(5\times10^{6}\text{ m/s})^2 \sin^2 45°}{2(1.6\times10^{-19}\text{ C})(3.5\times10^{3}\text{ N/C})} = 1.02\text{ cm}$$

and, because the plates are separated by 2 cm, the electron does not strike the upper plate.

To determine where the electron will strike the lower plate, set $y = 0$ in equation (1) and solve for x to obtain:

$$x = \frac{m_e v_0^2 \sin 2\theta}{eE_y}$$

Substitute numerical values and evaluate x:

$$x = \frac{\left(9.11\times10^{-31}\text{ kg}\right)\left(5\times10^{6}\text{ m/s}\right)^2 \sin 90^\circ}{\left(1.6\times10^{-19}\text{ C}\right)\left(3.5\times10^{3}\text{ N/C}\right)} = \boxed{4.07\text{ cm}}$$

85 •••

Picture the Problem We can use Coulomb's force law for point masses and the condition for translational equilibrium to express the equilibrium position as a function of k, q, Q, m, and g. In part (*b*) we'll need to show that the displaced point charge experiences a linear restoring force and, hence, will exhibit simple harmonic motion.

(*a*) Apply the condition for translational equilibrium to the point mass:

$$\frac{kqQ}{y_0^2} - mg = 0$$

Solve for y_0 to obtain:

$$y_0 = \boxed{\sqrt{\frac{kqQ}{mg}}}$$

(*b*) Express the restoring force that acts on the point mass when it is displaced a distance Δy from its equilibrium position:

$$F = \frac{kqQ}{(y_0 + \Delta y)^2} - \frac{kqQ}{y_0^2} \approx \frac{kqQ}{y_0^2 + 2y_0\Delta y} - \frac{kqQ}{y_0^2}$$

because $\Delta y << y_0$.

Simplify this expression further by writing it with a common denominator:

$$F = -\frac{2y_0 \Delta y kqQ}{y_0^4 + 2y_0^3\Delta y} = -\frac{2y_0 \Delta y kqQ}{y_0^4\left(1 + 2\frac{\Delta y}{y_0}\right)} \approx -\frac{2\Delta y kqQ}{y_0^3}$$

again, because $\Delta y << y_0$.

From the 1st step of our solution:

$$\frac{kqQ}{y_0^2} = mg$$

Substitute to obtain:

$$F = -\frac{2mg}{y_0}\Delta y$$

Apply Newton's 2nd law to the displaced point charge to obtain:

$$m\frac{d^2\Delta y}{dt^2} = -\frac{2mg}{y_0}\Delta y$$

or

$$\boxed{\frac{d^2\Delta y}{dt^2} + \frac{2g}{y_0}\Delta y = 0}$$

the differential equation of simple harmonic motion with $\boxed{\omega = \sqrt{2g/y_0}}$.

Ch 22 The Electric Field: Continuous Charge Distribution

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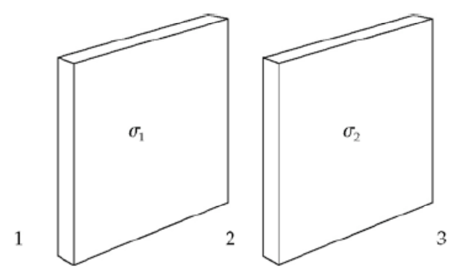
Picture the Problem Let the charge densities on the two plates be σ_1 and σ_2 and denote the three regions of interest as 1, 2, and 3. Choose a coordinate system in which the positive *x* direction is to the right. We can apply the equation for $\vec{E}$ near an infinite plane of charge and the superposition of fields to find the field in each of the three regions.



(*a*) Use the equation for $\vec{E}$ near an infinite plane of charge to express the field in region 1 when $\sigma_1 = \sigma_2 = +3\ \mu\text{C/m}^2$:

$$\begin{aligned}\vec{E}_1 &= \vec{E}_{\sigma_1} + \vec{E}_{\sigma_2} \\ &= -2\pi k\sigma_1\hat{i} - 2\pi k\sigma_2\hat{i} \\ &= -4\pi k\sigma\,\hat{i}\end{aligned}$$

Substitute numerical values and evaluate $\vec{E}_1$:

$$\vec{E}_1 = -4\pi\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3\ \mu\text{C/m}^2\right)\hat{i} = \boxed{-\left(3.39\times10^5\ \text{N/C}\right)\hat{i}}$$

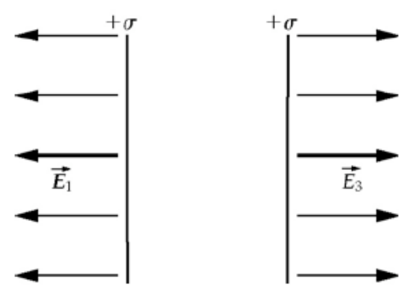
Proceed as above for region 2:

$$\begin{aligned}\vec{E}_2 &= \vec{E}_{\sigma_1} + \vec{E}_{\sigma_2} = 2\pi k\sigma_1\hat{i} - 2\pi k\sigma_2\hat{i} \\ &= 2\pi k\sigma\hat{i} - 2\pi k\sigma\hat{i} = \boxed{0}\end{aligned}$$

Proceed as above for region 3:

$$\begin{aligned}\vec{E}_3 &= \vec{E}_{\sigma_1} + \vec{E}_{\sigma_2} = 2\pi k\sigma_1\hat{i} + 2\pi k\sigma_2\hat{i} \\ &= 4\pi k\sigma\hat{i} \\ &= 4\pi\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3\ \mu\text{C/m}^2\right)\hat{i} \\ &= \boxed{\left(3.39\times10^5\ \text{N/C}\right)\hat{i}}\end{aligned}$$

The electric field lines are shown to the right:



(*b*) Use the equation for $\vec{E}$ near an infinite plane of charge to express and evaluate the field in region 1 when $\sigma_1 = +3\ \mu\text{C/m}^2$ and $\sigma_2 = -3\ \mu\text{C/m}^2$:

$$\begin{aligned}\vec{E}_1 &= \vec{E}_{\sigma_1} + \vec{E}_{\sigma_2} = 2\pi k\sigma_1\hat{i} - 2\pi k\sigma_2\hat{i} \\ &= 2\pi k\sigma\hat{i} - 2\pi k\sigma\hat{i} = \boxed{0}\end{aligned}$$

Proceed as above for region 2:

$$\begin{aligned}\vec{E}_1 &= \vec{E}_{\sigma_1} + \vec{E}_{\sigma_2} = 2\pi k\sigma_1\hat{i} + 2\pi k\sigma_2\hat{i} \\ &= 4\pi k\sigma\hat{i} \\ &= 4\pi\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3\ \mu\text{C}\right)\hat{i} \\ &= \boxed{\left(3.39\times10^5\ \text{N/C}\right)\hat{i}}\end{aligned}$$

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Picture the Problem The magnitude of the electric field on the axis of a ring of charge is given by $E_x(x) = kQx/(x^2 + a^2)^{3/2}$ where Q is the charge on the ring and a is the radius of the ring. We can use this relationship to find the electric field on the x axis at the given distances from the ring.

Express $\vec{E}$ on the axis of a ring charge:

$$E_x(x) = \frac{kQx}{(x^2 + a^2)^{3/2}}$$

(a) Substitute numerical values and evaluate E_x for x = 1.2 cm:

$$E_x(1.2\,\text{cm}) = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.75\,\mu\text{C})(1.2\,\text{cm})}{\left[(1.2\,\text{cm})^2 + (8.5\,\text{cm})^2\right]^{3/2}} = \boxed{4.69\times10^5\ \text{N/C}}$$

(b) Proceed as in (a) with x = 3.6 cm:

$$E_x(3.6\,\text{cm}) = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.75\,\mu\text{C})(3.6\,\text{cm})}{\left[(3.6\,\text{cm})^2 + (8.5\,\text{cm})^2\right]^{3/2}} = \boxed{1.13\times10^6\ \text{N/C}}$$

(c) Proceed as in (a) with x = 4.0 m:

$$E_x(4\,\text{m}) = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.75\,\mu\text{C})(4\,\text{m})}{\left[(4\,\text{m})^2 + (8.5\,\text{cm})^2\right]^{3/2}} = \boxed{1.54\times10^3\ \text{N/C}}$$

(d) Using Coulomb's law for the electric field due to a point charge, express E_x:

$$E_x(x) = \frac{kQ}{x^2}$$

*42 ••

Determine the Concept The charges on a conducting sphere, in response to the repulsive Coulomb forces each experiences, will separate until electrostatic equilibrium conditions exit. The use of a wire to connect the two spheres or to ground the outer sphere will cause additional redistribution of charge.

(*a*) Because the outer sphere is conducting, the field in the thin shell must vanish. Therefore, $-2Q$, uniformly distributed, resides on the inner surface, and $-5Q$, uniformly distributed, resides on the outer surface.

(*b*) Now there is no charge on the inner surface and $-5Q$ on the outer surface of the spherical shell. The electric field just outside the surface of the inner sphere changes from a finite value to zero.

(*c*) In this case, the $-5Q$ is drained off, leaving no charge on the outer surface and $-2Q$ on the inner surface. The total charge on the outer sphere is then $-2Q$.

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Picture the Problem The electric field is directed radially outward. We can construct a Gaussian surface in the shape of a cylinder of radius r and length L and apply Gauss's law to find the electric field as a function of the distance from the centerline of the infinitely long, uniformly charged cylindrical shell.

(*a*) Apply Gauss's law to a cylindrical surface of radius r and length L that is concentric with the inner conductor:

$$\oint_S E_n dA = \frac{1}{\epsilon_0} Q_{\text{inside}}$$

or

$$2\pi rLE_n = \frac{Q_{\text{inside}}}{\epsilon_0}$$

where we've neglected the end areas because no flux crosses them.

Solve for E_n:

$$E_n = \frac{2kQ_{\text{inside}}}{Lr} \qquad (1)$$

For $r < 1.5$ cm, $Q_{\text{inside}} = 0$ and:

$$E_n(r < 1.5\,\text{cm}) = \boxed{0}$$

Letting $R = 1.5$ cm, express Q_{inside} for 1.5 cm < r < 4.5 cm:

$$Q_{\text{inside}} = \lambda L = 2\pi\sigma RL$$

Substitute in equation (1) to obtain:

$$E_n(1.5\,\text{cm} < r < 4.5\,\text{cm}) = \frac{2k(\lambda L)}{Lr} = \frac{2k\lambda}{r}$$

Substitute numerical values and evaluate $E_n(1.5\text{ cm} < r < 4.5\text{ cm})$:

$$E_n(1.5\,\text{cm} < r < 4.5\,\text{cm}) = 2(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\frac{(6\,\text{nC/m})}{r} = \boxed{\frac{(108\,\text{N}\cdot\text{m/C})}{r}}$$

Express Q_{inside} for 4.5 cm < r < 6.5 cm:

$Q_{\text{inside}} = 0$

and

$E_n(4.5\,\text{cm} < r < 6.5\,\text{cm}) = \boxed{0}$

Letting σ_2 represent the charge density on the outer surface, express Q_{inside} for r > 6.5 cm:

$Q_{\text{inside}} = \sigma_2 A_2 = 2\pi\sigma_2 R_2 L$

where R_2 = 6.5 cm.

Substitute in equation (1) to obtain:

$$E_n(r > R_2) = \frac{2k(2\pi\sigma_2 R_2 L)}{Lr} = \frac{\sigma_2 R_2}{\epsilon_0\, r}$$

In (*b*) we show that σ_2 = 21.2 nC/m^2. Substitute numerical values to obtain:

$$E_n(r > 6.5\,\text{cm}) = \frac{(21.2\,\text{nC/m}^2)(6.5\,\text{cm})}{(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)r} = \boxed{\frac{156\,\text{N}\cdot\text{m/C}}{r}}$$

(*b*) The surface charge densities on the inside and the outside surfaces of the outer conductor are given by:

$$\sigma_1 = \frac{-\lambda}{2\pi R_1} \text{ and } \sigma_2 = -\sigma_1$$

Substitute numerical values and evaluate σ_1 and σ_2:

$$\sigma_1 = \frac{-6\,\text{nC/m}}{2\pi(0.045\,\text{m})} = \boxed{-21.2\,\text{nC/m}^2}$$

and

$$\sigma_2 = \boxed{21.2\,\text{nC/m}^2}$$

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Picture the Problem We can integrate the density function over the radius of the inner cylinder to find the charge on it and then calculate the linear charge density from its definition. To find the electric field for all values of r we can construct a Gaussian surface in the shape of a cylinder of radius r and length L and apply Gauss's law to each region of the cable to find the electric field as a function of the distance from its centerline.

(*a*) Find the charge Q_{inner} on the inner cylinder:

$$Q_{\text{inner}} = \int_0^R \rho(r)dV = \int_0^R \frac{C}{r}2\pi rL\,dr$$

$$= 2\pi CL\int_0^R dr = 2\pi CLR$$

Relate this charge to the linear charge density:

$$\lambda_{\text{inner}} = \frac{Q_{\text{inner}}}{L} = \frac{2\pi CLR}{L} = 2\pi CR$$

Substitute numerical values and evaluate λ_{inner}:

$$\lambda_{\text{inner}} = 2\pi(200\,\text{nC/m})(0.015\,\text{m}) = \boxed{18.8\,\text{nC/m}}$$

(*b*) Apply Gauss's law to a cylindrical surface of radius r and length L that is concentric with the infinitely long nonconducting cylinder:

$$\oint_S E_n dA = \frac{1}{\epsilon_0} Q_{\text{inside}}$$

or

$$2\pi r L E_n = \frac{Q_{\text{inside}}}{\epsilon_0}$$

where we've neglected the end areas because no flux crosses them.

Solve for E_n:

$$E_n = \frac{Q_{\text{inside}}}{2\pi r L \epsilon_0}$$

Substitute to obtain, for $r < 1.5$ cm:

$$E_n(r < 1.5\,\text{cm}) = \frac{2\pi C L r}{2\pi \epsilon_0 L r} = \frac{C}{\epsilon_0}$$

Substitute numerical values and evaluate $E_n(r < 1.5\text{ cm})$:

$$E_n(r < 1.5\,\text{cm}) = \frac{200\,\text{nC/m}^2}{8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2} = \boxed{22.6\,\text{kN/C}}$$

Express Q_{inside} for $1.5\text{ cm} < r < 4.5\text{ cm}$:

$$Q_{\text{inside}} = 2\pi CLR$$

Substitute to obtain, for $1.5\text{ cm} < r < 4.5\text{ cm}$:

$$E_n(1.5\,\text{cm} < r < 4.5\,\text{cm}) = \frac{2C\pi RL}{2\pi \epsilon_0 rL} = \frac{CR}{\epsilon_0 r}$$

where $R = 1.5$ cm.

Substitute numerical values and evaluate $E_n(1.5\text{ cm} < r < 4.5\text{ cm})$:

$$E_n(1.5\,\text{cm} < r < 4.5\,\text{cm}) = \frac{(200\,\text{nC/m}^2)(0.015\,\text{m})}{(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)r} = \boxed{\frac{339\,\text{N}\cdot\text{m/C}}{r}}$$

Because the outer cylindrical shell is a conductor:

$$E_n(4.5\,\text{cm} < r < 6.5\,\text{cm}) = \boxed{0}$$

For $r > 6.5$ cm, $Q_{\text{inside}} = 2\pi CLR$ and:

$$E_n(r > 6.5\,\text{cm}) = \boxed{\frac{339\,\text{N}\cdot\text{m/C}}{r}}$$

Ch 23 Electric Potential

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Picture the Problem We can use $\vec{E} = -\frac{\partial V}{\partial x}\hat{i}$ to find the electric field corresponding the given potential and then compare its form to those produced by the four alternatives listed.

Find the electric field corresponding to this potential function:

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} = -\frac{\partial}{\partial x}\left[4|x| + V_0\right]\hat{i}$$

$$= -4\frac{\partial}{\partial x}\left[|x|\right]\hat{i} = -4\begin{bmatrix} 1\text{ if } x \geq 0 \\ -1\text{ if } x < 0 \end{bmatrix}\hat{i}$$

$$= \begin{bmatrix} -4\text{ if } x \geq 0 \\ 4\text{ if } x < 0 \end{bmatrix}\hat{i}$$

Of the alternatives provided above, only a uniformly charged sheet in the *yz* plane would produce a constant electric field whose direction changes at the origin. $\boxed{(c)\text{ is correct.}}$

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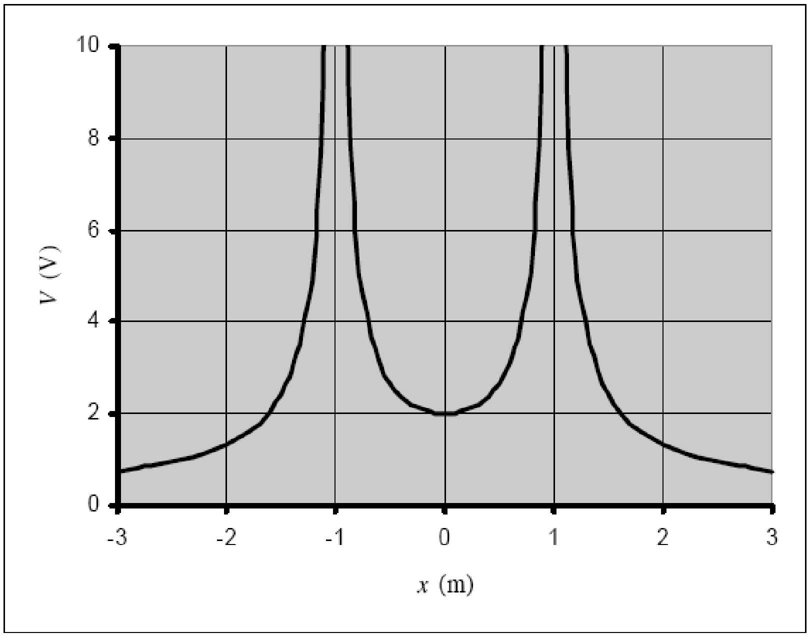
Picture the Problem For the two charges, $r = |x-a|$ and $|x+a|$ respectively and the electric potential at x is the algebraic sum of the potentials at that point due to the charges at $x = +a$ and $x = -a$.

(*a*) Express $V(x)$ as the sum of the potentials due to the charges at $x = +a$ and $x = -a$:

$$V = \boxed{kq\left(\frac{1}{|x-a|} + \frac{1}{|x+a|}\right)}$$

(*b*) The following graph of $V(x)$ versus x for $kq = 1$ and $a = 1$ was plotted using a spreadsheet program:



(*c*) At $x = 0$:

$$\frac{dV}{dx} = \boxed{0} \text{ and } E_x = -\frac{dV}{dx} = \boxed{0}$$

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Picture the Problem We can find the potential on the x axis at $x = 3.00$ m by expressing it as the sum of the potentials due to the charges at the origin and at $x = 6$ m. We can also express the Coulomb field on the x axis as the sum of the fields due to the charges q_1 and q_2 located at the origin and at $x = 6$ m.

(*a*) Express the potential on the x axis as the sum of the potentials due to the charges q_1 and q_2 located at the origin and at $x = 6$ m:

$$V(x) = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

Substitute numerical values and evaluate V(3 m):

$$V(x)=\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\times\left(\frac{3\,\mu\text{C}}{3\,\text{m}}+\frac{-3\,\mu\text{C}}{3\,\text{m}}\right)=\boxed{0}$$

(b) Express the Coulomb field on the x axis as the sum of the fields due to the charges q_1 and q_2 located at the origin and at
x = 6 m:

$$E_x=\frac{kq_1}{r_1^2}+\frac{kq_2}{r_2^2}=k\left(\frac{q_1}{r_1^2}+\frac{q_2}{r_2^2}\right)$$

Substitute numerical values and evaluate E(3 m):

$$E_x=\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\times\left(\frac{3\,\mu\text{C}}{(3\,\text{m})^2}-\frac{-3\,\mu\text{C}}{(3\,\text{m})^2}\right)=\boxed{5.99\,\text{kV/m}}$$

(c) Express the potential on the x axis as the sum of the potentials due to the charges q_1 and q_2 located at the origin and at
x = 6 m:

$$V(x)=k\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}\right)$$

Substitute numerical values and evaluate V(3.01 m):

$$V(3.01\,\text{m})=\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\times\left(\frac{3\,\mu\text{C}}{3.01\,\text{m}}+\frac{-3\,\mu\text{C}}{2.99\,\text{m}}\right)=\boxed{-59.9\,\text{V}}$$

Compute $-\Delta V/\Delta x$:

$$-\frac{\Delta V}{\Delta x}=-\frac{-59.9\,\text{V}-0}{3.01\,\text{m}-3.00\,\text{m}}=\boxed{5.99\,\text{kV/m}}=E_x(3.00\,\text{m})$$

44 •

Picture the Problem We can construct Gaussian surfaces just inside and just outside the spherical shell and apply Gauss's law to find the electric field at these locations. We can use the expressions for the electric potential inside and outside a spherical shell to find the potential at these locations.

(*a*) Apply Gauss's law to a spherical Gaussian surface of radius r < 12 cm:

$$\oint_S \vec{E}\cdot d\vec{A}=\frac{Q_{\text{enclosed}}}{\epsilon_0}=0$$

because the charge resides on the outer surface of the spherical surface. Hence $\vec{E}(r<12\,\text{cm})=\boxed{0}$

Apply Gauss's law to a spherical Gaussian surface of radius

$$E\left(4\pi r^2\right)=\frac{q}{\epsilon_0}$$

$r > 12$ cm: and

$$E(r > 12\,\text{cm}) = \frac{q}{4\pi r^2 \in_0} = \frac{kq}{r^2}$$

Substitute numerical values and evaluate $E(r > 12\,\text{cm})$:

$$E(r > 12\,\text{cm}) = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10^{-8}\ \text{C})}{(0.12\,\text{m})^2} = \boxed{6.24\,\text{kV/m}}$$

(b) Express and evaluate the potential just inside the spherical shell:

$$V(r \le R) = \frac{kq}{R} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10^{-8}\ \text{C})}{0.12\,\text{m}} = \boxed{749\,\text{V}}$$

Express and evaluate the potential just outside the spherical shell:

$$V(r \ge R) = \frac{kq}{r} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10^{-8}\ \text{C})}{0.12\,\text{m}} = \boxed{749\,\text{V}}$$

(c) The electric potential inside a uniformly charged spherical shell is constant and given by:

$$V(r \le R) = \frac{kq}{R} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10^{-8}\ \text{C})}{0.12\,\text{m}} = \boxed{749\,\text{V}}$$

In part (a) we showed that: $\vec{E}(r < 12\,\text{cm}) = \boxed{0}$

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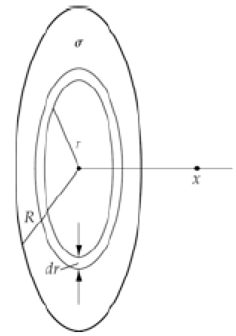
Picture the Problem We can find Q by integrating the charge on a ring of radius r and thickness dr from $r = 0$ to $r = R$ and the potential on the axis of the disk by integrating the expression for the potential on the axis of a ring of charge between the same limits.



(a) Express the charge dq on a ring of radius r and thickness dr:

$$dq = 2\pi r\sigma dr = 2\pi r\left(\sigma_0 \frac{R}{r}\right)dr$$
$$= 2\pi\sigma_0 R dr$$

Integrate from $r = 0$ to $r = R$ to obtain:

$$Q = 2\pi\sigma_0 R\int_0^R dr = \boxed{2\pi\sigma_0 R^2}$$

(b) Express the potential on the axis of the disk due to a circular element of charge $dq = 2\pi r\sigma dr$:

$$dV = \frac{kdq}{r'} = \frac{2\pi k\sigma_0 R dr}{\sqrt{x^2 + r^2}}$$

Integrate from $r = 0$ to $r = R$ to obtain:

$$V = 2\pi k\sigma_0 R\int_0^R \frac{dr}{\sqrt{x^2 + r^2}}$$
$$= \boxed{2\pi k\sigma_0 R\ln\left(\frac{R + \sqrt{x^2 + R^2}}{x}\right)}$$

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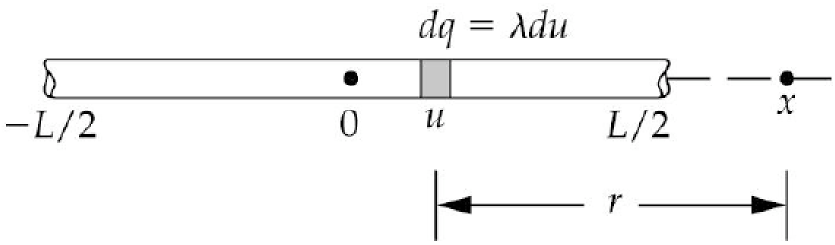
Picture the Problem We can express the electric potential dV at x due to an elemental charge dq on the rod and then integrate over the length of the rod to find $V(x)$. In the second part of the problem we use a binomial expansion to show that, for $x >> L/2$, our result reduces to that due to a point charge Q.



(*a*) Express the potential at x due to the element of charge dq located at u:

$$dV = \frac{kdq}{r} = \frac{k\lambda du}{x-u}$$

or, because $\lambda = Q/L$,

$$dV = \frac{kQ}{L}\frac{du}{x-u}$$

Integrate V from $u = -L/2$ to $L/2$ to obtain:

$$V(x) = \frac{kQ}{L}\int_{-L/2}^{L/2}\frac{du}{x-u}$$

$$= \frac{kQ}{L}\ln(x-u)\Big|_{-L/2}^{L/2}$$

$$= \left[-\ln\left(x-\frac{L}{2}\right)+\ln\left(x+\frac{L}{2}\right)\right]$$

$$= \boxed{\frac{kQ}{L}\ln\left(\frac{x+\frac{L}{2}}{x-\frac{L}{2}}\right)}$$

(*b*) Divide the numerator and denominator of the argument of the logarithm by x to obtain:

$$\ln\left(\frac{x+\frac{L}{2}}{x-\frac{L}{2}}\right) = \ln\left(\frac{1+\frac{L}{2x}}{1-\frac{L}{2x}}\right) = \ln\left(\frac{1+a}{1-a}\right)$$

where $a = L/2x$.

Divide $1 + a$ by $1 - a$ to obtain:

$$\ln\left(\frac{1+a}{1-a}\right) = \ln\left(1+2a+\frac{2a^2}{1-a}\right)$$

$$= \ln\left(1+\frac{L}{x}+\frac{\frac{L^2}{x^2}}{2-\frac{L}{x}}\right)$$

$$\approx \ln\left(1+\frac{L}{x}\right)$$

provided $x >> L/2$.

Expand $\ln(1 + L/x)$ binomially to obtain:

$$\ln\left(1+\frac{L}{x}\right) \approx \frac{L}{x}$$

provided $x >> L/2$.

Substitute to express $V(x)$ for $x >> L/2$:

$$V(x) = \frac{kQ}{L}\frac{L}{x} = \boxed{\frac{kQ}{x}}$$, the field due to a

point charge Q.

Ch 24 Electrostatic Energy and Capacitance

48 ••

Picture the Problem We can use the expression for the capacitance of a parallel-plate capacitor to find the area of each plate and the definition of capacitance to find the potential difference when the capacitor is charged to 3.2 μC. We can find the stored energy using $U = \frac{1}{2}CV^2$ and the definition of capacitance and the relationship between the potential difference across a parallel-plate capacitor and the electric field between its plates to find the charge at which dielectric breakdown occurs. Recall that $E_{\text{max, air}} = 3$ MV/m.

(*a*) Relate the capacitance of a parallel-plate capacitor to the area A of its plates and their separation d:

$$C = \frac{\epsilon_0 A}{d}$$

Solve for A:

$$A = \frac{Cd}{\epsilon_0}$$

Substitute numerical values and evaluate A:

$$A = \frac{(0.14\,\mu\text{F})(0.5\,\text{mm})}{8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2} = \boxed{7.91\,\text{m}^2}$$

(*b*) Using the definition of capacitance, express and evaluate the potential difference across the capacitor when it is charged to 3.2 μC:

$$V = \frac{Q}{C} = \frac{3.2\,\mu\text{C}}{0.14\,\mu\text{F}} = \boxed{22.9\,\text{V}}$$

(*c*) Express the stored energy as a function of the capacitor's capacitance and the potential difference across it:

$$U = \tfrac{1}{2}CV^2$$

Substitute numerical values and evaluate U:

$$U = \tfrac{1}{2}(0.14\,\mu\text{F})(22.9\,\text{V})^2 = \boxed{36.7\,\mu\text{J}}$$

(*d*) Using the definition of capacitance, relate the charge on the capacitor to breakdown potential difference:

$$Q_{\text{max}} = CV_{\text{max}}$$

Relate the maximum potential difference to the maximum electric field between the plates:

$$V_{\text{max}} = E_{\text{max}}d$$

Substitute to obtain:

$$Q_{\text{max}} = CE_{\text{max}}d$$

Substitute numerical values and evaluate Q_{max}:

$$Q_{\text{max}} = (0.14\,\mu\text{F})(3\,\text{MV/m})(0.5\,\text{mm}) = \boxed{210\,\mu\text{C}}$$

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Picture the Problem The capacitance of a cylindrical capacitor is given by $C = 2\pi\kappa \in_0 L/\ln(r_2/r_1)$ where L is its length and r_1 and r_2 the radii of the inner and outer conductors.

(*a*) Express the capacitance of the coaxial cylindrical shell:

$$C = \frac{2\pi\kappa \in_0 L}{\ln\left(\frac{r}{R}\right)}$$

Substitute numerical values and evaluate C:

$$C = \frac{2\pi(1)\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)(0.12\ \mathrm{m})}{\ln\left(\frac{1.5\ \mathrm{cm}}{0.2\ \mathrm{mm}}\right)} = \boxed{1.55\ \mathrm{pF}}$$

(*b*) Use the definition of capacitance to express the charge per unit length:

$$\lambda = \frac{Q}{L} = \frac{CV}{L}$$

Substitute numerical values and evaluate λ:

$$\lambda = \frac{(1.55\ \mathrm{pF})(1.2\ \mathrm{kV})}{0.12\ \mathrm{m}} = \boxed{15.5\ \mathrm{nC/m}}$$

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Picture the Problem When the capacitors are reconnected, each will have the charge it acquired while they were connected in series across the 12-V battery and we can use the definition of capacitance and their equivalent capacitance to find the common potential difference across them. In part (*b*) we can use $U = \frac{1}{2}CV^2$ to find the initial and final energy stored in the capacitors.

(*a*) Using the definition of capacitance, express the potential difference across each capacitor when they are reconnected:

$$V = \frac{2Q}{C_{\mathrm{eq}}} \qquad (1)$$

where Q is the charge on each capacitor *before* they are disconnected.

Find the equivalent capacitance of the two capacitors after they are connected in parallel:

$$C_{eq} = C_1 + C_2 = 4\,\mu\text{F} + 12\,\mu\text{F} = 16\,\mu\text{F}$$

Express the charge Q on each capacitor before they are disconnected:

$$Q = C'_{eq}V$$

Express the equivalent capacitance of the two capacitors connected in series:

$$C'_{eq} = \frac{C_1C_2}{C_1 + C_2} = \frac{(4\,\mu\text{F})(12\,\mu\text{F})}{4\,\mu\text{F} + 12\,\mu\text{F}} = 3\,\mu\text{F}$$

Substitute to find Q:

$$Q = (3\,\mu\text{F})(12\,\text{V}) = 36\,\mu\text{C}$$

Substitute in equation (1) and evaluate V:

$$V = \frac{2(36\,\mu\text{C})}{16\,\mu\text{F}} = \boxed{4.50\,\text{V}}$$

(*b*) Express and evaluate the energy stored in the capacitors initially:

$$U_i = \tfrac{1}{2}C'_{eq}V_i^2 = \tfrac{1}{2}(3\,\mu\text{F})(12\,\text{V})^2 = \boxed{216\,\mu\text{J}}$$

Express and evaluate the energy stored in the capacitors when they have been reconnected:

$$U_f = \tfrac{1}{2}C_{eq}V_f^2 = \tfrac{1}{2}(16\,\mu\text{F})(4.5\,\text{V})^2 = \boxed{162\,\mu\text{J}}$$

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Picture the Problem Let the numeral 1 denote the 35-μF capacitor and the numeral 2 the 10-μF capacitor. We can use $U = \frac{1}{2}C_{eq}V^2$ to find the energy initially stored in the system and the definition of capacitance to find the charges on the two capacitors. When the dielectric is removed from the capacitor the two capacitors will share the total charge stored equally. Finally, we can find the final stored energy from the total charge stored and the equivalent capacitance of the two equal capacitors in parallel.

(*a*) Express the stored energy of the system in terms of the equivalent capacitance and the charging potential:

$$U = \tfrac{1}{2}C_{eq}V^2$$

Express the equivalent capacitance:

$$C_{eq} = C_1 + C_2$$

Substitute to obtain:

$$U = \tfrac{1}{2}(C_1 + C_2)V^2$$

Substitute numerical values and evaluate U:

$$U = \tfrac{1}{2}(35\,\mu\text{F} + 10\,\mu\text{F})(100\,\text{V})^2 = \boxed{0.225\,\text{J}}$$

(*b*) Use the definition of capacitance to find the charges on the two capacitors:

$$Q_1 = C_1V = (35\,\mu\text{F})(100\,\text{V}) = \boxed{3.50\,\text{mC}}$$

and

$$Q_2 = C_2V = (10\,\mu\text{F})(100\,\text{V}) = \boxed{1.00\,\text{mC}}$$

(*c*) Because the capacitors are connected in parallel, when the dielectric is removed their charges will be equal; as will be their capacitances and:

$$Q_1 = Q_2 = \tfrac{1}{2}Q = \tfrac{1}{2}(3.5\,\text{mC} + 1\,\text{mC}) = \boxed{2.25\,\text{mC}}$$

(*d*) Express the final stored energy in terms of the total charge stored and the equivalent capacitance:

$$U_f = \frac{1}{2}\frac{Q_{tot}^2}{Q_{eq}}$$

Substitute numerical values and evaluate U_f:

$$U_f = \frac{1}{2}\frac{(4.5\,\text{mC})^2}{2(10\,\mu\text{F})} = \boxed{0.506\,\text{J}}$$

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Picture the Problem We can use the equations for the equivalent capacitance of capacitors connected in parallel and in series to find the single capacitor that will store the same amount of charge as each of the networks shown above.

(*a*) Find the capacitance of the two capacitors in parallel:

$$C_{\text{eq},1} = C_0 + C_0 = 2C_0$$

Find the capacitance equivalent to $2C_0$ in series with C_0:

$$C_{\text{eq},2} = \frac{C_{\text{eq},1}C_0}{C_{\text{eq},1} + C_0} = \frac{(2C_0)C_0}{2C_0 + C_0} = \boxed{\tfrac{2}{3}C_0}$$

(*b*) Find the capacitance of two capacitors of capacitance C_0 in parallel:

$$C_{\text{eq},1} = 2C_0$$

Find the capacitance equivalent to $2C_0$ in series with $2C_0$:

$$C_{\text{eq},2} = \frac{C_{\text{eq},1}C_0}{C_{\text{eq},1} + C_0} = \frac{(2C_0)(2C_0)}{2C_0 + 2C_0} = \boxed{C_0}$$

(*c*) Find the equivalent capacitance of three equal capacitors connected in parallel:

$$C_{\text{eq}} = C_0 + C_0 + C_0 = \boxed{3C_0}$$

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Picture the Problem We can treat the configuration in (*a*) as two capacitors in parallel and the configuration in (*b*) as two capacitors in series. Finding the equivalent capacitance of each configuration and examining their ratio will allow us to decide whether (*a*) or (*b*) has the greater capacitance. In both cases, we'll let C_1 be the capacitance of the dielectric-filled capacitor and C_2 be the capacitance of the air capacitor.

In configuration (*a*) we have: $$C_a = C_1 + C_2$$

Express C_1 and C_2:

$$C_1 = \frac{\kappa \in_0 A_1}{d_1} = \frac{\kappa \in_0 \frac{1}{2} A}{d} = \frac{\kappa \in_0 A}{2d}$$

and

$$C_2 = \frac{\in_0 A_2}{d_2} = \frac{\in_0 \frac{1}{2} A}{d} = \frac{\in_0 A}{2d}$$

Substitute for C_1 and C_2 and simplify to obtain:

$$C_a = \frac{\kappa \in_0 A}{2d} + \frac{\in_0 A}{2d} = \frac{\in_0 A}{2d}(\kappa + 1)$$

In configuration (b) we have:

$$\frac{1}{C_b} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_b = \frac{C_1 C_2}{C_1 + C_2}$$

Express C_1 and C_2:

$$C_1 = \frac{\in_0 A_1}{d_1} = \frac{\in_0 A}{\frac{1}{2} d} = \frac{2 \in_0 A}{d}$$

and

$$C_2 = \frac{\kappa \in_0 A_2}{d_2} = \frac{\kappa \in_0 A}{\frac{1}{2} d} = \frac{2\kappa \in_0 A}{d}$$

Substitute for C_1 and C_2 and simplify to obtain:

$$C_b = \frac{\left(\frac{2 \in_0 A}{d}\right)\left(\frac{2\kappa \in_0 A}{d}\right)}{\frac{2 \in_0 A}{d} + \frac{2\kappa \in_0 A}{d}} = \frac{\left(\frac{2 \in_0 A}{d}\right)\left(\frac{2\kappa \in_0 A}{d}\right)}{\frac{2 \in_0 A}{d}(\kappa + 1)} = \frac{2 \in_0 A}{d}\left(\frac{\kappa}{\kappa + 1}\right)$$

Divide C_b by C_a:

$$\frac{C_b}{C_a} = \frac{\frac{2 \in_0 A}{d}\left(\frac{\kappa}{\kappa + 1}\right)}{\frac{\in_0 A}{2d}(\kappa + 1)} = \frac{4\kappa}{(\kappa + 1)^2}$$

Because $\frac{4\kappa}{(\kappa + 1)^2} < 1$ for $\kappa > 1$:

$$\boxed{C_a > C_b}$$