Homework # 5 Solutions

1. (a) The magnetic force on the proton is
   \[ F = qvB \sin \theta \]
   .

   (b) Its acceleration is
   \[ a = \frac{F}{m_p} = \frac{qvB \sin \theta}{m_p} \].

2. (a) The electron accelerates through a potential difference \( \Delta V \), giving it kinetic energy
   \[ |q_e| \Delta V = \frac{1}{2} m_e v^2 \],
   such that its speed \( v \) is
   \[ v = \sqrt{\frac{2|q_e| \Delta V}{m_e}} \].
   The maximum value of the force is when the velocity is perpendicular to the B field, in which case
   \[ |F| = |q_e v B| \].

   (b) The minimum value of the force occurs when the velocity is parallel to the B field, in which case \( F = 0 \).

3. Given the velocity \( \vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k} \) and the magnetic field \( \vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k} \), the magnetic force on the proton is
   \[ \vec{F} = q_p \vec{v} \times \vec{B} = (v_y B_z - v_z B_y) \hat{i} + (v_z B_x - v_x B_z) \hat{j} + (v_x B_y - v_y B_x) \hat{k} \].
   The magnitude of the force is \( |\vec{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2} \).

4. We are given that the velocity of the electron is \( \vec{v} = v_e \hat{i} \), its acceleration is \( \vec{a} = a_e \hat{k} \), and the electric field is \( \vec{E} = E \hat{k} \). The components of the magnetic field can be determined using the Lorentz force law:
   \[ \vec{F} = m_e \vec{a} = q_e \vec{E} + q_e \vec{v} \times \vec{B} \],
   which in this case takes the form
   \[ m_e a_e \hat{k} = q_e E \hat{k} + q_e v_e \hat{i} \times \vec{B} \].
   Since there is no acceleration in the \( y \) direction, we can conclude that \( B_z = 0 \). Furthermore, \( B_x \) is undetermined since it has a vanishing contribution to the magnetic force (since \( B_x \) is parallel to the
velocity). The \( y \) component of the magnetic field will provide a magnetic force in the \( z \) direction. The Lorentz force law becomes
\[
m_e a_e = q_e E + q_e v_e B_y,
\]
such that the \( y \) component of the magnetic field is
\[
B_y = \frac{1}{v_e} \left( -\frac{m_e a_e}{|q_e|} - E \right).
\]

5. The rod experiences a magnetic force \( F = IBd \) directed toward the right. The work done as it rolls a distance \( L \) is equal to the change in its kinetic energy. Since the rod is rolling without slipping, the kinetic energy equals the translational kinetic energy \( \frac{1}{2}mv^2 \) plus the rotational kinetic energy \( \frac{1}{2}I_{\text{rod}}\omega^2 \), where \( \omega = v/R \) is the angular velocity, and the moment of inertia \( I_{\text{rod}} = \frac{1}{2}mR^2 \). Therefore, the work-energy relation takes the form
\[
IBdL = \frac{1}{2}mv^2 + \frac{1}{2}I_{\text{rod}}\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2} \left( \frac{1}{2}mR^2 \right) \left( \frac{v}{R} \right)^2 = \frac{3}{4}mv^2,
\]
such that
\[
v = \sqrt{\frac{4IBdL}{3m}}.
\]

6. (a) The magnetic moment of the current loop of circumference \( 2\pi r \) is
\[
\mu = IA = I\pi r^2.
\]
(b) The torque exerted by the magnetic field on the loop is
\[
|\vec{\tau}| = |\vec{\mu} \times \vec{B}| = |\mu B|.
\]

7. For the singly charged ion, the velocity is obtained from
\[
q\Delta V = \frac{1}{2}mv^2,
\]
such that
\[
v = \sqrt{\frac{2q\Delta V}{m}}.
\]
The radius of its semicircular orbit is
\[
R = \frac{mv}{qB} = \frac{1}{B} \sqrt{\frac{2\Delta V}{q}}.
\]

\[2\]
If the radius of the orbit of a multiply charged ion with $q' = nq$ and mass $m'$ is $R' = NR$, we have the relation

$$\frac{R'}{R} = N = \sqrt{\frac{m' q}{m q'}} = \sqrt{\frac{1}{n} \frac{m'}{m}},$$

such that

$$\frac{m'}{m} = N^2 n.\]}

8. The velocity selected in the velocity selector is

$$v = \frac{E}{B}.\]}

The radius of the circular orbit of a singly charged ion (i.e., $q = q_p = 1.6 \times 10^{-19}$ C) in the mass spectrometer is

$$r = \frac{mv}{qB} = \frac{mE}{qB^2}.\]}

9. (a) The Hall voltage is related to the magnetic field as follows:

$$\Delta V_H = \frac{IB}{nqt}.\]}

For the Hall probe with current $I$, the initial data for the Hall voltage and the magnetic field allows us to solve for $(nqt)^{-1}$:

$$\frac{1}{nqt} = \frac{\Delta V_H}{IB}.\]}

We can then use Eq. (1) to solve for the new magnetic field $B'$ given a new Hall voltage $\Delta V'_H$:

$$B' = nqt \frac{\Delta V'_H}{I} = B \frac{\Delta V'_H}{\Delta V_H}.\]}

(b) To determine $n$, use Eq. (1):

$$n = \frac{1}{qt} \frac{IB}{\Delta V_H}.\]}

10. The magnetic field a distance $r$ away from a long current-carrying wire is

$$B = \frac{\mu_0 I}{2\pi r}.\]