

# **Physics 202 Exam 1**

## **Oct 3, 2006**

Name: Key

Section:

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Instructions:

1. Don't forget to write down your name and section number.
2. Only one answer is allowed for each multiple-choice question.
3. **Except for the multiple-choice problems, a reasonable amount of work is required to receive full credit.**
4. Be aware that intermediate steps earn points even if the final answer is incorrect.
5. Erase (or cross out) any mistakes or you will be marked down. Grading is based on everything you have written down.
6. Sig. figure rules are not tested in this exam.

Scores:

**Problem 1** \_\_\_\_\_

**Problem 2** \_\_\_\_\_

**Problem 3** \_\_\_\_\_

**Problem 4** \_\_\_\_\_

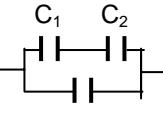
**Problem 5** \_\_\_\_\_

**Problem 6** \_\_\_\_\_

**Total** \_\_\_\_\_

**Problem 1: multiple-choice questions (30 points, 3 points each):**

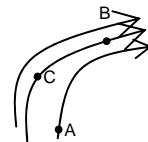
Please circle the correct answer for each question below. Only one correct answer per question is expected. Answering with more than one choice earns zero credit.

- (1) If both charge A and charge B repel charge C electrically, then
  - (a) A and B attract each other electrically.
  - (b) A and B repel each other electrically.
  - (c) Not sure, it depends on the sign of charge C.
  - (d) Not sure, it depends on the sign of charge A and B.
  
- (2) Two (unconnected) conductors are in electrostatic equilibrium.
  - (a) The total charge on the two conductors must be zero.
  - (b) The force between the conductors must be zero.
  - (c) The electric potentials of them must be equal.
  - (d) None of above is necessarily true.
  
- (3) In a static electric field, electric potential is higher at location A than at location B. Which of the following statements is true?
  - (a) A positive test charge is subject to a larger electric force at A than at B.
  - (b) A negative test charge is subject to a larger electric force at A than at B.
  - (c) A positive test charge has higher electric potential energy at A than at B.
  - (d) A negative test charge has higher electric potential energy at A than at B.
  
- (4) Three capacitors,  $C_1=1\mu F$ ,  $C_2=2\mu F$  and  $C_3=3\mu F$  are connected as shown.  


The combined capacitance is:

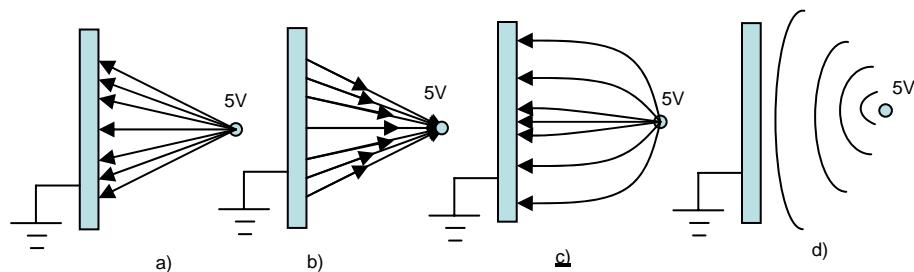
  - (a)  $6\mu F$
  - (b)  $3\mu F$
  - (c)  $1.5\mu F$
  - (d) None of above.
  
- (5) Two conducting spheres A and B, of radii R and  $3R$ , respectively, are electrically connected by a thin conducting wire. Assume potential  $V=0$  at infinity. What is the ratio of potentials on spheres ( $V_A:V_B$ )?
  - (a) 1:3
  - (b) 1:9
  - (c) 9:1
  - (d) None of above.

- (6) Two capacitors,  $C_1$  and  $C_2=2C_1$  are connected in series. When charged,
- The energy stored in  $C_1$  is twice that in  $C_2$ .
  - The energy stored in  $C_1$  is half that in  $C_2$ .
  - The energy stored in  $C_1$  is the same as that in  $C_2$ .
  - Not sure, it depends on the charges on the capacitors.



- (7) A negative test charge is to be placed in the electric field shown.  
At which position, A, B, or C, the test charge has the highest potential energy?
- A
  - B
  - C
  - Not sure, it depends on the field strength at each location.

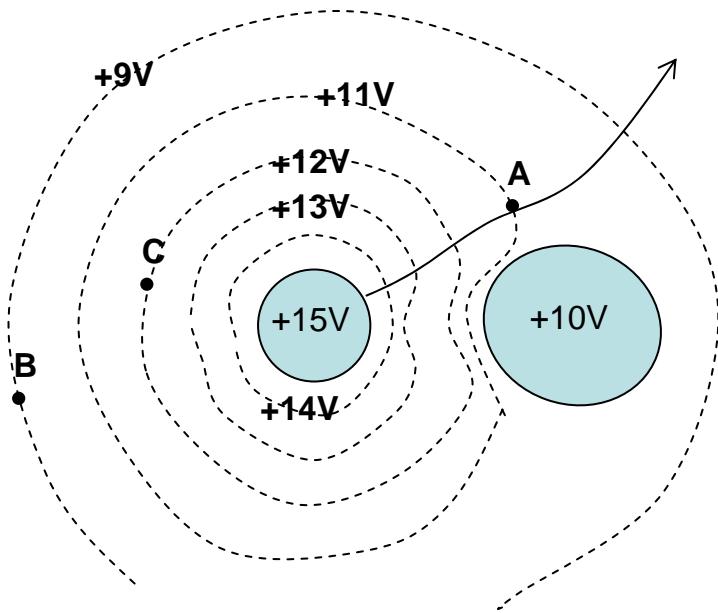
- (8) Which of the following statements is true:
- Gauss's law is valid only when there is no charge outside the Gaussian surface.
  - Gauss's law is valid only when the Gaussian surface is highly symmetric.
  - Gauss's law is valid only when the Gaussian surface is highly symmetric and there is no charge outside the Gaussian surface.
  - None of above.
- (9) Which of the following figures best represents the field lines between a +5V electrode and a nearby grounded conducting plate.



- (10) A point charge  $Q=-0.6\text{mC}$  is released from rest at point A in an electric field. When it reaches point B, its kinetic energy is 7.2J. What is the potential difference  $V_B - V_A$ ?
- 12 kV.
  - 12 kV.
  - Can not be determined as the mass of the charge is not given.
  - None of above.

**Problem 2 (15 points):** Two conductors are connected to +15V and +10V, respectively. The equipotential lines of the field are shown in the figure below.

- Draw on the figure the electric field line passing point A. (5 points)
- For a negative charge, at which position, A or B, is its potential energy higher. (5 points)
- What is the external work required to move a test charge of  $1\mu\text{C}$  from point A to point C. Ignore the effect on the electric field due to the test charge. (5 points)



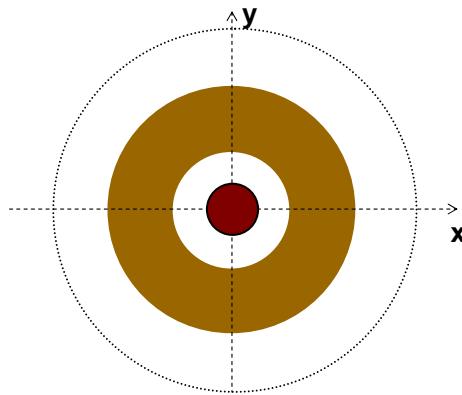
Solutions:

- See Fig.
- Location B
- $W = q(V_f - V_i) = 1\mu\text{C} * (12\text{V} - 11\text{V}) = 1\mu\text{J}$

**Problem 3 (20 points):** A conducting shell, of inner radius  $R$  and outer radius  $2R$ , carries a total charge  $-2Q$  ( $Q > 0$ ). A solid conducting sphere, of radius  $0.5R$  and carrying a charge of  $-Q$ , is placed at the center of the shell.

(Note: your answers have to come with supporting arguments. A simply copy of formulas will not earn full credit.)

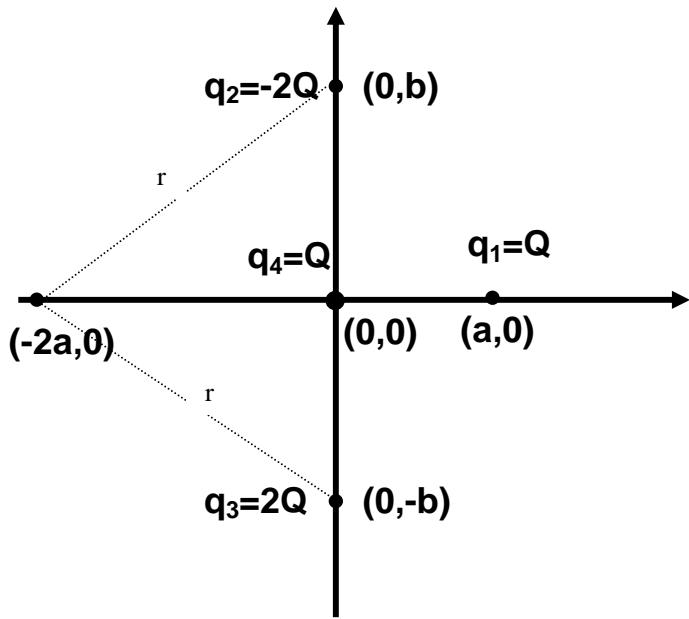
- Find the electric field at  $x=0.25R$  ( 5 points )
- Find the electric field at  $x=3R$ . ( 5 points )
- What is the surface charge density on the outer surface of conducting shell? (i.e. what is the surface charge density at  $2R$ ? ) ( 5 points )
- Compare( $>$ ,  $<$ ,  $=$ ) the electric potential at  $x=0.25R$  and  $x=1.5R$ . ( 5 points)



Solutions:

- $E=0$  as there is no field inside a conductor
- Draw a Gaussian sphere at  $3R$  as shown. By symmetry, the field is radial with same magnitude in all directions.  
 $\Phi_{3R} = 4\pi(3R)^2 E = q_{in}/\epsilon_0 \rightarrow E = 1/(12\pi \epsilon_0) Q/R^2$  pointing towards center.
- Center sphere carries charge  $-Q \rightarrow$  inner surface of the shell carries total charge of  $+Q$   
 $\rightarrow q_{outer\_surface\_at\_2R} = -3Q \rightarrow s2R = q/A = -3Q/(4\pi R^2) = -3Q/(16\pi R^2)$
- $V_{0.25R} = V_{0.5R}$ ,  $V_{1.5R} = V_{1.0R}$ , and between  $0.5R$  and  $1.0R$  the field lines point towards center.  $\rightarrow V_{0.5R} < V_{1.0R} \rightarrow V_{0.25R} < V_{1.5R}$

**Problem 4 (10 points):** Four point charges,  $q_1, q_2, q_3, q_4$ , of charges  $Q, -2Q, 2Q$ , and  $Q$ , respectively, are placed in locations  $(a,0)$ ,  $(0,b)$ ,  $(0,-b)$  and  $(0,0)$  as shown.



- What is the electric force on  $q_4$ ? (5 points)
- Now  $q_4$  is moved from  $(0,0)$  to  $(-2a,0)$ , how much is the work done by the electric forces due to other three (fixed) charges during the process? (5 points)

**Solutions:**

a)  $F_4 = F_{14} + F_{24} + F_{34}$

$$F_{14} = k_e q_1 q_4 / a^2 (-\hat{i}) = k_e Q^2 / a^2 (-\hat{i}),$$

$$F_{24} = k_e q_2 q_4 / b^2 (-\hat{j}) = k_e 2Q^2 / b^2 (\hat{j}), \quad F_{34} = k_e q_3 q_4 / b^2 (\hat{j}) = k_e 2Q^2 / b^2 (\hat{j})$$

$$F_4 = k_e Q^2 / a^2 (-\hat{i}) + k_e 4Q^2 / b^2 (\hat{j})$$

b)  $W = -q_4 (V_{(-2a,0)} - V_{(0,0)})$

$$V_{(0,0)} = k_e q_1 / a + k_e q_2 / b + k_e q_3 / b = k_e Q / a \quad (\text{note: contributions from } q_2 \text{ and } q_3 \text{ cancel})$$

$$V_{(-2a,0)} = k_e q_1 / 3a + k_e q_2 / r + k_e q_3 / r = k_e Q / 3a \quad (\text{note: contributions from } q_2 \text{ and } q_3 \text{ cancel})$$

$$W = -Q(k_e Q / 3a - k_e Q / a) = 2/3 k_e Q^2 / a$$

**Problem 5 (15 points):** A parallel plate capacitor, with separation  $d$  between the plates, is initially connected to a battery of voltage  $V$ . The electrical energy stored in the capacitor is  $U_0$ .

- a) What is the charge on the capacitor? (5 points)
- b) What is the stored electrical energy after the separation between the plates is increased to  $2d$  while the battery remains connected? (5 points).
- c) Now remove the battery and start to reduce the gap between the plates. What is the stored electrical energy once the separation is reduced back to  $d$  (from  $2d$ )? (5 points)

Solutions:

Let  $C_d$  and  $C_{2d}$  be the capacitance when separation is  $d$  and  $2d$ , respectively. Note:  $C \sim 1/d$

a)  $U_0 = \frac{1}{2} C_d V^2 = \frac{1}{2} QV \rightarrow Q = 2U_0/V$

b) As the battery remains connected, the potential difference is unchanged.

$$U = \frac{1}{2} CV^2 \text{ and } C_{2d} = \frac{1}{2} C_d \rightarrow U_{2d} = \frac{1}{2} U_0$$

c) Once the battery is removed, the charge on the capacitor is unchanged.

$$U = \frac{1}{2} Q^2/C, C_d = 2C_{2d} \rightarrow U_d = \frac{1}{2} U_{2d} = \frac{1}{4} U_0$$

**Problem 6** (10 points): The electric potential of an electric field is described by  $V=y^2+3x^2y+yz$ . (SI units implied)

Among three locations in the field,  $(x,y,z)=(0,0,0)$ ,  $(1,1,1)$ , and  $(1,0,0)$ :

- At which location is the electrical energy density highest? (5 points).
- At which location is a negative test charge would have highest electrical potential energy. (5 points)

Solutions:

a)  $u_E = \frac{1}{2} \epsilon_0 E^2$ , so all we need is to find out  $E$ .

$$E_x = -\frac{\partial V}{\partial x} = -6xy, E_y = -\frac{\partial V}{\partial y} = -(3x^2 + 2y + z), E_z = -\frac{\partial V}{\partial z} = -y.$$

$$\text{at } (0,0,0): E_x = E_y = E_z = 0 \rightarrow E = 0$$

$$\text{at } (1,1,1): E_x = -6, E_y = -6, E_z = -1 \rightarrow E = \sqrt{73}$$

$$\text{at } (1,0,0): E_x = 0, E_y = -3, E_z = 0 \rightarrow E = 3$$

→  $(1,1,1)$  has highest energy density among the three.

b)  $V_{(0,0,0)} = 0, V_{(1,1,1)} = 5, V_{(1,0,0)} = 0$ ,

Higher potential means lower potential energy for a negative test charge.

→ At either  $(0,0,0)$  or  $(1,0,0)$ , the negative test charge has highest potential energy.