Physics 202 Midterm Exam 3
November 26, 2007

Name: ...................................................  ID#: ............................................

Section: ......................

TA (please circle):
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Instructions:

1. Don’t forget to write down your name and section number.

2. Show your work! A reasonable amount of work is required to receive full credit.

3. Be aware that intermediate steps earn points even if the final answer is incorrect.

4. Erase (or cross out) any mistakes or you will be marked down. Grading is based on everything you have written down.

5. Both the magnitude and direction of vector quantities need to be specified for full credit.

Fundamental Constants:

\[ \varepsilon_0 = (4\pi k_e)^{-1} = 8.85 \times 10^{-12} \text{C}^2/(\text{N} \cdot \text{m}^2) \quad \mu_0 = 4\pi \times 10^{-7} \text{T} \cdot \text{m} / \text{A} \quad c = 3 \times 10^8 \text{m/s} \]
\[ m_p = 1.67 \times 10^{-27} \text{kg} \quad m_e = 9.11 \times 10^{-31} \text{kg} \quad q_p = -q_e = 1.6 \times 10^{-19} \text{C} \]

Scores:

Problem 1 25  Problem 2 15  Problem 3 20
Problem 4 20  Problem 5 20
Problem 1: (25 points)
A conducting loop of width w and length d is moved with constant speed v to the right. It passes through a uniform magnetic field B as illustrated.

a) Indicate the direction of the induced current (if any) in the figure above for all three positions.

b) Sketch the following:
   - the magnetic flux through the area enclosed by the loop versus x (sign convention: positive flux into the page, negative flux out of the page). Take the right edge of the loop as reference (see the dotted line; i.e., take x=0 when the loop enters the field):

   ![Magnetic Flux Graph](image)

   - the induced motional emf versus x:

   ![Induced EMF Graph](image)

   - the external force applied necessary to keep the speed v constant:

   ![External Force Graph](image)
In the example above, assume that the resistor is 6.00 Ω, and that the magnetic field has the magnitude 2.50 T and is directed perpendicularly into the paper. Let d = 1.20 m. The loop is pulled at a constant speed of 2 m/s.

c) What is the induced current when the loop enters the field?

\[ \mathcal{E} = -\frac{d \Phi}{dt} \quad \Phi = A \cdot B = B \cdot d \cdot x \]

\[ \mathcal{E} = -B \cdot d \cdot v \]

\[ I = \frac{\mathcal{E}}{R} = -\frac{BDv}{R} = -1 \text{A} \]

\[ I = -1 \text{A} \]

5

d) Calculate the applied force required to move the bar to the right at this speed.

\[ \mathbf{F} = B \cdot \mathbf{d} \cdot I \]

\[ \mathbf{F} = 3 \text{N} \]

5

e) If the resistor is only 0.0006 Ω, what would be the resulting speed, if everything else is kept the same.

\[ \mathbf{F} = B \cdot \mathbf{d} \cdot I = \frac{B^2 d^2 v}{R} \]

\[ \mathbf{F}, B, d \text{ same} \]

\[ \rightarrow v \sim \frac{1}{R} \]

\[ R_2 = 10^{-4} R_e \]

\[ \nu = 2 \cdot 10^{-4} \frac{\text{m}}{5} \]

5
Problem 2:

Consider the circuit shown in the figure. Let $L = 10.00 \, \text{H}$, $R = 10.00 \, \Omega$, and $\varepsilon = 100 \, \text{V}$.

a) What is the time constant of the circuit?

$$\tau = \frac{L}{R}$$

b) - What is the current at the time $t = 0$, immediately after the switch is closed?

- What is the current after a long time?

- Provide a sketch of the current versus time.

$$I = \frac{\varepsilon}{R} (1 - e^{-t/\tau})$$

$$I_{\text{max}} = \frac{\varepsilon}{R} = 10 \, \text{A}$$

5 points

c) Provide a sketch of the voltage across $R$ and across $L$ versus time.

$$\Delta V_R = \varepsilon (1 - e^{-t/\tau})$$

$$\Delta V_L = \varepsilon e^{-t/\tau}$$

4 points

d) At what time is the voltage across the resistor exactly 50V?

$$\Delta V_R(t) = \varepsilon (1 - e^{-t/\tau}) = 50 \, \text{V}$$

$$1 - e^{-t/\tau} = 0.5$$

$$- \frac{t}{\tau} = \ln 0.5$$

$$t = - \tau \cdot \ln 0.5 = 0.693 \, \text{s}$$

5 points
Problem 3
A radar transmitter contains a serial RLC circuit. The capacitor is adjusted such that the circuit is in resonance at a certain frequency f.

a) Which of the following statements is correct? Circle the correct answers.

2 + n correct answers \rightarrow n points

i. The impedance of the circuit is maximal at this frequency. [YES / NO]

ii. The voltage of the capacitor is in phase with the voltage across the inductor. [YES / NO]

iii. The voltage across the inductor leads the current by 90°. [YES / NO]

iv. The voltage across the capacitor leads the current by 90°. [YES / NO]

v. There is no power dissipated in the circuit. [YES / NO]

vi. The energy stored in L and C combined does not change with time. [YES / NO]

vii. The capacitive and the inductive reactance are the same. [YES / NO]

viii. The impedance of the circuit is zero at this frequency. [YES / NO]

In the following, let L = 400 pH, R = 10 Ω. The resonance frequency \(f_0 = 1.0 \times 10^{10}\) Hz.

b) What is the reactance of the capacitor C at this frequency?

\[ X_C = \frac{1}{\omega C} = X_L = \omega L = \frac{2\pi f L}{L} \]

\[ X_C = 25.1 \Omega \]

3 points

c) Determine the impedance of the circuit at resonance.

\[ Z = \sqrt{R^2 + (X_L - X_C)^2} = R \]

\[ Z(\omega_0) = 10 \Omega \]

3 points

d) Now we double the frequency to \(f = 2f_0\) (nothing else changes). What is the impedance in this case?

\[ X_C = \frac{1}{2} \times 25.1 \Omega \]

\[ X_L = 2 \times 25.1 \Omega \]

\[ X_L - X_C = \frac{3}{2} \times 25.1 \Omega \]

\[ Z = \sqrt{100 + \left(\frac{3}{2} \times 25.1\right)^2} \Omega \]

\[ Z = 39.0 \Omega \]

4 points

e) Draw a to-scale phasor diagram for the RLC circuit at the new frequency and estimate the phase angle from the diagram.

\[ \phi \approx 75° \] (between 60° and 90°)

4 points
Problem 4:
The wave function for a wave on a taut string is given below, where \( x \) is in meters and \( t \) is in seconds.

\[
y(x, t) = (0.5 \text{ m}) \sin(8t - 2x + \pi/4)
\]

a) What is the wavelength of the wave?

\[
k = \frac{2\pi}{\lambda} \quad \lambda = \frac{2\pi}{k}
\]

b) What is the speed of this wave on the string?

\[
u = \frac{\omega}{k} = \]

\[
u = 4 \frac{m}{s}
\]

c) What is the transverse velocity of an element of the string at \( t=1 \) s and \( x=3 \) m?

\[
u_y = \frac{dy}{dt} = 4 \cdot \cos \left( 8t - 2x + \frac{\pi}{4} \right) \frac{m}{s}
\]

\[
u_y (t=1, x=3) = 4 \cdot \cos \left( 8 - 6 + \frac{\pi}{4} \right) \frac{m}{s} = -3.75 \frac{m}{s}
\]

d) The tension on the string is 10 N. What is the mass density of the string?

\[
u = \sqrt{\frac{T}{\mu}} \rightarrow \nu^2 = \frac{T}{\mu}
\]

\[
\mu = \frac{T}{\nu^2} = 0.625 \frac{kg}{m}
\]

e) If a 1.5 m long wire of the same mass density as above is connected to a wall on one side and is connected via a pulley to a mass of 2 kg on the other side, what is the fundamental frequency of vibration?

\[
\lambda = 2 \ell = 3 \text{ m}
\]

\[
\ell = 1.5 \text{ m}
\]

\[
u = \sqrt{\frac{T}{\mu}} = \lambda f
\]

\[
f = \frac{1}{2\ell} \sqrt{\frac{T}{\mu}} = \frac{1}{3} \sqrt{\frac{2.98}{0.625}}
\]

\[
f = 1.87 \text{ Hz}
\]
Problem 5:
A sinusoidal electromagnetic wave of frequency 30 MHz travels in free space in the x direction.

a) Determine the wavelength \( \lambda \) of this wave.
\[
\lambda = \frac{c}{f}
\]
\[
\lambda = 10 \text{ m}
\]

b) We observe that the electric field has a maximum amplitude of \( E_{\text{max}} = 100 \text{ V/m} \) at \( t=0 \), \( x=5 \text{ m} \).
Determine the wave function \( B(x,t) \) of this wave.
\[
B(x,t) = 0.33 \cdot 10^{-6} \text{ T} \cdot \cos(0.63x - 1.8 \cdot 10^{-6}t - \pi)
\]

\[
\Phi = -\pi \quad \text{need a phase factor}
\]

\[
\begin{align*}
\kappa &= \frac{2\pi}{\lambda} \\
\omega &= 2\pi f \\
B_{\text{max}} &= \frac{E_{\text{max}}}{c} = \frac{1}{3} \cdot 10^{-6} \text{ T}
\end{align*}
\]

\[
5 \text{ m} = \frac{1}{2} \lambda 
\]

\[
\lambda = 5 \text{ m} 
\]

\[
\begin{align*}
\text{c) What is the force due to the radiation pressure that this wave will generate on a surface of area } & 30 \text{ m}^2 \text{ that reflects 40\% of the incoming radiation?} \\
P &= \frac{S}{c} \text{ for absorption} \\
S_{\text{ave}} &= I = \frac{E_{\text{max}}^2}{2\mu_0 c} \\
\text{Pressure} \\
P &= 1.4 \cdot \frac{S_{\text{ave}}}{c} \\
F &= \rho \cdot A
\end{align*}
\]
\[
F = 1.86 \cdot 10^{-6} \text{ N}
\]