Physics 202
Chapter 32
Oct 25, 2007

- Induction:
  - Inductance,
  - Self-Inductance,
  - RL circuit
  - Energy of B-field

On whiteboard

- RL circuit
  - Solve Kirchhoff’s equation for the case of
  - Switching on, in detail
  - Switching off
- Power
- Energy of B-field, dissipation in R
Demonstrations

- **Eddy currents**
  - Conducting plate entering localized strong B-field
  - Discussion on whiteboard (remember the difference to a plate moving inside a homogenous field!)
  - Spherical magnet rolling slowly inside copper tube
  - Magnet bouncing of a thick cold conducting copper plate because of Eddy currents

- **RL circuit:** voltage and current when switching on and off

Self-Inductance

- When the switch is closed, the current does not immediately reach its maximum value
- Faraday’s law can be used to describe the effect:
  - Close switch: $I$ increases, $dI/dt \neq 0$
  - $\Rightarrow \Phi_B$ increases
  - $\Rightarrow$ Self-induced emf, according to Faraday’s law
  - $\Rightarrow$ Induced current in opposite direction
  - $\Rightarrow$ Net current is reduced.

\[
e = -\frac{d\Phi_B}{dt} \propto \frac{dI}{dt}
\]

\[
\varepsilon_L = -L \frac{dI}{dt}
\]
Self-Inductance, Coil Example

- A current in the coil produces a magnetic field directed toward the left (a)
- If the current increases, the increasing flux creates an induced emf of the polarity shown (b)
- The polarity of the induced emf reverses if the current decreases (c)

Self-Inductance

- A induced emf is always proportional to the time rate of change of the current
- \( L \) := inductance
  - a measure of the opposition to a change in current
  - \( L \) depends on the specifics of the coil
- Symbol for an inductor:

\[
\varepsilon_L = -L \frac{dI}{dt}
\]

\[
L = -\frac{\varepsilon_L}{dI/dt}
\]

- Inductance of a solenoid on the whiteboard

\[
1H = 1 \frac{V \cdot s}{A}
\]

Unit: Henry (named for Joseph Henry)
**RL Circuit, Analysis**

- An *RL* circuit contains an inductor and a resistor.

- When the switch is closed (at time *t* = 0), the current begins to increase.

- At the same time, a back emf is induced in the inductor that opposes the change, - the increasing current.

\[ I = \frac{\varepsilon}{R} \left(1 - e^{-\frac{Rt}{L}}\right) \]

where \( \tau = \frac{L}{R} \)
**RL Circuit**

- Applying Kirchhoff’s loop rule:
  \[-IR - L \frac{dI}{dt} = 0\]

- This one is very easy to solve. Separation of variables and integration (whiteboard) yields:
  \[I = \frac{E}{R} e^{-\frac{R}{L}t} = \frac{E}{R} e^{-\frac{1}{\tau}t}\]
  where \(\tau = \frac{L}{R}\)

**Energy in a Magnetic Field**

- In a circuit with an inductor, the battery must supply more energy than in a circuit without an inductor.
- Part of the energy supplied by the battery appears as internal energy in the resistor.
- The remaining energy is stored in the magnetic field of the inductor.

\[U = \frac{1}{2} L \cdot I^2\]

- Compare to the energy stored in the electric field in a capacitor.

\[U = \frac{1}{2} C \cdot (\Delta V)^2\]
**Energy density of the Magnetic Field**

- Energy stored in the magnetic field:
  - Just as energy can be stored in the electric field in a capacitor

\[ u_B = \frac{B^2}{2\mu_0} \]

- The energy density of the electric field:

\[ u_E = \frac{1}{2} \varepsilon_0 E^2 \]