Physics 202, Lecture 4

Today’s Topics

- Review: Gauss’s Law
- Electric Potential (Ch. 25-Part I)
  - Electric Potential Energy and Electric Potential
  - Electric Potential and Electric Field
- Next Tuesday: Electric Potential (Ch. 25-Part II)
- Homework #1 due tomorrow (9/14) at 10 PM
  Homework #2 (now on WebAssign) due 9/24 at 10 PM

Gauss’s Law: Review

\[ \Phi_E = \oint E \cdot d\mathbf{A} = \frac{\sum q_{in}}{\varepsilon_0} \]

Fundamental equation of electrostatics (equivalent to Coulomb’s Law)

Can use it to obtain E for highly symmetric charge distributions.
Method: evaluate flux over carefully chosen “Gaussian surface”:

- **spherical** (point chg, uniform sphere, spherical shell, ...)
- **cylindrical** (infinite uniform line of charge or cylinder, ...)
- **planar** (infinite uniform sheet of charge, ...)
Gauss’s Law: Examples

1. **Spherical symmetry** (last lecture).

2. **Cylindrical symmetry.** Example: infinite uniform line of charge.

   Symmetry: E indep of \( z, \theta \), in radial direction

   Gaussian surface: cylinder of length \( L \)

   \[
   \oint \vec{E} \cdot d\vec{A} = E(r)2\pi rL = \frac{q_{in}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0} \]

   \( \vec{E}(r) = \frac{\lambda}{2\pi \varepsilon_0 r} \hat{r} \)

3. **Planar symmetry.** Example: infinite uniform sheet of charge.

   Symmetry: E indep of \( x,y \), in z direction

   Gaussian surface: pillbox, area of faces=\( A \)

   \[
   \oint \vec{E} \cdot d\vec{A} = 2EA = \frac{q_{in}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} \Rightarrow \vec{E} = \frac{\sigma}{2\varepsilon_0} \hat{z} \]

   Exercise: try for E field just outside of a conductor

Electric Potential Energy and Electric Potential

**Review: Conservation of Energy (particle)**

- **Kinetic Energy (K)**
  \[
  K = \frac{1}{2}mv^2
  \]

- **Potential Energy \( U \): conservative forces (work independent of path)**
  \[
  U(x,y,z)
  \]

- If only conservative forces present in system, conservation of mechanical energy: \( K + U = \) constant

- **Examples of conservative forces:**
  - Springs: elastic potential energy \( U = k_{spring}x^2/2 \)
  - Gravity: gravitational potential energy
  - Electrostatic: electric potential energy (today)

- **Examples of nonconservative forces**
  - Friction, viscous damping (terminal velocity)
Electric Potential Energy (I)

Compare with gravitational force (Ch. 13):

\[ \vec{F}_{12} = -G \frac{m_1 m_2}{r^2} \hat{r}_{12} \]

\[ W = \int_{\text{path}} \vec{F} \cdot d\vec{s} = \frac{G m_1 m_2}{r_f} - \frac{G m_1 m_2}{r_i} \]

\[ \Rightarrow \text{Gravitational Potential energy:} \]

\[ U = -\frac{G m_1 m_2}{r} \]

\[ \Rightarrow \text{Electric Force:} \]

\[ \vec{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{r}_{12} \]

\[ \Rightarrow \text{Electric Potential Energy} \]

\[ U = \frac{k_e q_1 q_2}{r} \]

Electric Potential Energy (II)

Given two positive charges \( q \) and \( q_0 \):

Initially charges very far apart: \( U_i = 0 \)

(we are free to define the potential energy zero somewhere)

To push particles together requires work (they want to repel).

Final potential energy will increase! \( \Delta U = U_f - U_i = \Delta W \)

Now, suppose \( q \) is fixed at the origin. What is work required to move \( q_0 \) from infinity to a distance \( r \) away from \( q \)?

\[ \Delta W = \int_{\infty}^{r} \vec{F}_{us} \cdot d\vec{s} = -\int_{\infty}^{r} \vec{F}_e \cdot d\vec{s} = -\int_{\infty}^{r} \frac{k_e q q_0}{r^2} \, dr' = \frac{k_e q q_0}{r} \]

Note: if \( q \) negative, final potential energy negative

Particles will move to minimize their final potential energy!
**Electric Potential Energy: Summary**

- Electric potential energy between two point charges:
  \[ U(r) = \frac{kq_0q}{r} \]
  - U is a scalar quantity, can be + or -
  - Convenient choice: U=0 at r= ∞
  - SI unit: Joule (J)

- Electric potential energy for system of multiple charges:
  - Sum over pairs:
    \[ U(r) = \sum_{i<j} kq_iq_j / r_{ij} \]
  - Integral if continuous distribution

**Example: Three Charge system**

- What is work required to assemble the three charge system as shown? \( q_1=q_2=q_3=Q \)
  - Answer: \( k_e \cdot 3Q^2/a \) (see board)

- What if \( q_1=q_2=Q \) but \( q_3=-Q \)?
  - Answer: \(-k_e \cdot Q^2/a\)
Electric Potential Energy: 
Charge In An Electric Field

- Charge $q_0$ is subject to Coulomb force in electric field $\mathbf{E}$:
  \[ \mathbf{F} = q_0 \mathbf{E} \]

- Work done by electric force:
  \[ W = \int_{i}^{f} \mathbf{F} \cdot d\mathbf{s} = q_0 \int_{i}^{f} \mathbf{E} \cdot d\mathbf{s} = -\Delta U \]

\[ \Delta U = U_f - U_i = -q_0 \int_{i}^{f} \mathbf{E} \cdot d\mathbf{s} \]

independent of $q_0$

Electric Potential Difference

- Electric Potential Energy: $q_0$ In a Generic E. Field
  \[ \Delta U = U_B - U_A = -q_0 \int_{A}^{B} \mathbf{E} \cdot d\mathbf{s} = q_0 \Delta V \]

- Electric Potential Difference
  \[ \Delta V \equiv \frac{\Delta U}{q_0} = -\int_{A}^{B} \mathbf{E} \cdot d\mathbf{s} = V_B - V_A \]
Properties of the Electric Potential

- Results from conservative nature of the electric force
- Associated with source field only (indep. of test charge)
- Units: J/C ≡ Volt (V)
- Often called potential, but meaningful only as potential difference \( V_B - V_A \).
  - A convenient point (∞, earth..) typically chosen as “ground” → \( \Delta V = V - (V_A = 0) = V \)
- Scalar quantity (no vector operations necessary!)
- Related to electric potential energy by \( \Delta U = q_0 \Delta V \)

Exercise 2: E. Potential and Point Charges

In the configuration shown, find \( V_B - V_A \)

Answer:

\[ V_B - V_A = k_e \left( \frac{q}{r_B} - \frac{q}{r_A} \right) \]

(See board)
Exercise 1: Uniform E. Field

In the uniform electric field shown:

1. Find potential at B, C, D, G

2. If a charge +q is placed at B, what is the potential energy $U_B$?

3. If now a $-q$ is at B, what is $U_B$?

4. If a $-q$ is initially at rest at G, will it move to A or B?

5. What is the kinetic energy when it reaches A?

A Picture to Remember

- Field lines always point towards lower electric potential
- In an electric field:
  - positive charges are always subject to a force in the direction of field lines, towards lower V
  - negative charge is always subject a force in the opposite direction of field lines, towards higher V
Obtaining the Electric Field From the Electric Potential

- Three ways to calculate the electric field
  - Coulomb’s Law $\mathbf{E} = \sum \mathbf{E}_i$
  - Gauss’s Law
  - Derive from electric potential
- Formalism

$$\Delta V = - \int_{A}^{B} \mathbf{E} \cdot d\mathbf{s}$$

$$dV = - \mathbf{E} \cdot d\mathbf{s} = -E_x dx - E_y dy - E_z dz$$

$$E_x = - \frac{\partial V}{\partial x}, \quad E_y = - \frac{\partial V}{\partial y}, \quad E_z = - \frac{\partial V}{\partial z} \quad \text{or} \quad \mathbf{E} = - \nabla V$$