Physics 202, Lecture 3

Today’s Topics

- Electric Field
  - Quick Review
  - Motion of Charged Particles in an Electric Field
- Gauss’s Law (Ch. 24, Serway)
- Conductors in Electrostatic Equilibrium (Ch. 24)
- Homework #2: on WebAssign tonight. Due 9/24, 10 PM

The Electric Field

1. **Charges are a source of electric fields.**

   Single point charge at origin: $\vec{E}(\vec{r}) = k_e \frac{q}{r^2} \hat{r}$

   Visualization: field lines

   Multiple charges: $\vec{E}(\vec{r}) = k_e \sum_i \frac{q_i (\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3}$

   Distributions: $\vec{E}(\vec{r}) = k_e \int dq (\vec{r} - \vec{r}') \frac{1}{|\vec{r} - \vec{r}'|^3}$
The Electric Field (II)

2. Charges respond to electric fields.

\[ \vec{F} = q \vec{E} \]
\[ \vec{a} = \frac{\vec{F}}{m} = \frac{q \vec{E}}{m} \]

Example: uniform E field, uniformly accel. motion!

\[ v = v_0 + at \]
\[ s = s_0 + v_0 t + \frac{1}{2} at^2 \]
\[ v^2 = v_0^2 + 2as \]

Exercise: Electron in Uniform E Field

- What is the vertical displacement after an electron passes through a region with uniform electric field \( \mathbf{E} \)?

- Solution: See board. Answer: \( dy = -\frac{1}{2} \left( \frac{|e|}{m} \right) E \left( \frac{r}{v_i} \right)^2 \)
Calculating E Fields

- We have seen how to calculate electric fields given a charge distribution (discrete or continuous)
  
  often very complicated to carry out analytically! one example: uniform sphere (from last lecture)

- A different but equivalent statement of Coulomb’s law:

  **Gauss’s Law**

  one of four fundamental equations of electromagnetism (Maxwell’s equations)

Electric Flux

- The electric flux $\Delta \Phi_E$ through an area element $\Delta A$:
  
  dot product of the electric field and the area vector:
  
  $\Delta \Phi_E = E \cdot \Delta A = E \Delta A \cos \theta$
  
  (area vector: normal to surface)

  $\Phi_E = \int E \cdot dA$

- Net electric flux through a closed surface:

  $\Phi_E = \oint E \cdot dA$

  Visualization:

  # field lines through surface
Flux through Closed Surfaces

- Compare fluxes through closed surfaces $s_1, s_2, s_3$: 
  \[ \Phi_{s1} = \Phi_{s2} = \Phi_{s3} \]
  
  (# field lines same through all 3 surfaces)

- Note: if no charge inside surface, $\Phi_{s1} = \Phi_{s2} = \Phi_{s3} = 0!$
  
  (# field lines going in = # field lines going out)

Gauss's Law (1)

- Gauss’s Law: net electric flux through any **closed** surface (“Gaussian surface”) equals the total charge enclosed inside the closed surface divided by the permittivity of free space.

  \[ E = \frac{\sum q_{in}}{\varepsilon_0} \]
  
  $q_{in}$: all charges enclosed regardless of positions

  \[ \varepsilon_0: \text{permittivity constant} \quad \left( 4\pi \varepsilon_0 \right)^{-1} = \kappa \]

  Gaussian surface (any shape)
Gauss’s Law (2)

\[ \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{in}}}{\varepsilon_0} \]

**Gauss’s Law:**

True in all situations, but not always easy to use...

However, Gauss’s Law is a powerful calculational tool in specific cases where the charge distribution exhibits a high degree of symmetry.

**Using Gauss’ Law**

To solve for the electric field using Gauss’s Law, it is necessary to choose a closed (Gaussian) surface such that the surface integral is trivial.

How to choose a Gaussian surface: use symmetry arguments.

1. **Direction.** Choose a surface such that \( \mathbf{E} \) is known to be either parallel or perpendicular to each piece of surface

2. **Magnitude.** Choose a surface such that \( \mathbf{E} \) is known to have the same value at all points on the surface

Then:

\[ \oint \mathbf{E} \cdot d\mathbf{A} = \oint E dA = E \oint dA = \frac{q_{\text{in}}}{\varepsilon_0} \]

Given \( q_{\text{in}} \), can solve for \( E \) (at surface), and vice versa
Last Lecture’s Example Again: Uniformly Charged Sphere

- A uniformly charged sphere radius $a$ and total charge $Q$, find electric field outside and inside the sphere.
- Solution: (see board, note the arguments on symmetry)

Apply Gauss’s Law Outside the Sphere

Apply Gauss’s Law inside the Sphere

Uniform Charge Sphere: Final Solution

<table>
<thead>
<tr>
<th>Inside</th>
<th>Outside</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E = \frac{kQ}{r^2}$</td>
<td>$E = \frac{kQ}{a^2}$</td>
</tr>
</tbody>
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Note: same form as a point charge
Another Example: Thin Spherical Shell

- Find E field inside/outside a uniformly charged thin spherical shell. Solution: see board.

Gaussian Surface for E(outside)  
Gaussian Surface for E(inside)

Result

Other examples: Be familiar with them!

- Infinite line of charge
- Infinite uniform charged sheet
- Infinite charged cylinder

Conductors And Electrostatic Equilibrium

- Conductors: charges (electrons) able to move freely  
  ➔ Charges redistribute when subject to E field.
- Charge redistribution ➔ electrostatic equilibrium.

Initial  ➔ transient, <10^{-16}s  
(right after E applied)  ➔ equilibrium
Properties of Electrostatic Equilibrium

- E field is always zero inside the conductor.
- E field on the surface of conductor:
  - normal to the surface, and magnitude \( E = \frac{\sigma}{\varepsilon_0} \)
  - (show using Gauss’s law)
- All charges reside on the surface of conductor.
- E field is also zero inside any cavity within the conductor.
  - (why?)

The above properties are valid regardless of the shape of and the total charge on the conductor!

How not to apply Gauss’s Law

- Two charges +2Q and –Q are placed at locations shown. Find the electric field at point P.
- 1. Draw a Gaussian surface passing P
  - 2. Apply Gauss’s law:
    \[ \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{in}}{\varepsilon_0} \]
    - \( q_{in} = +2Q + (-Q) = Q \)
  - 3. Surface integral:
    \[ \oint \mathbf{E} \cdot d\mathbf{A} = 4\pi r^2 E \]
    - \( E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \)

Is this correct? No! Which step is wrong? Last step
Example

- What is the electric flux through closed surface $S$?
  - $\Phi = 0$
  - $\Phi = (q_1 + q_2 + q_3 + q_4 + q_5)/\varepsilon_0$
  - $\Phi = (q_1 + q_2 + q_3)/\varepsilon_0$