# Optimizing QCA Performance*

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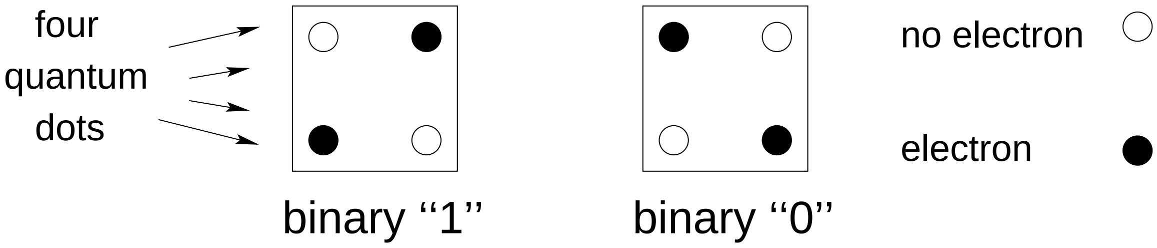
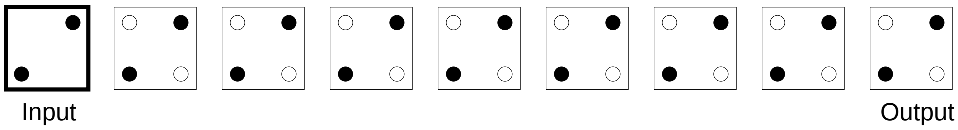
# Quantum-Dot Cellular Automata (QCA)

C. S. Lent *et al.*: [Nanotech. **4**, 49 (1993)]

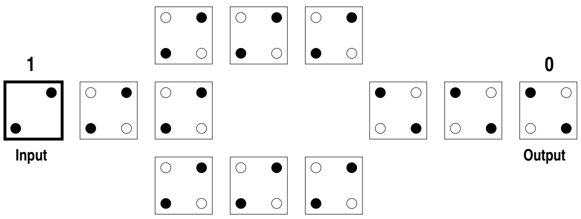



## QCA binary wire

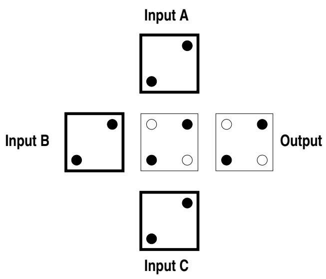



## QCA logic gates

**Inverter**




**Majority logic gate**

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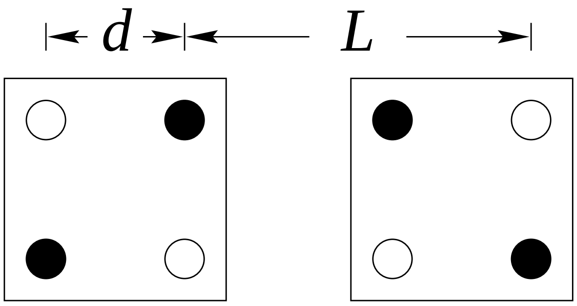
# Classical Electrostatic Model for QCA

Dimensions:

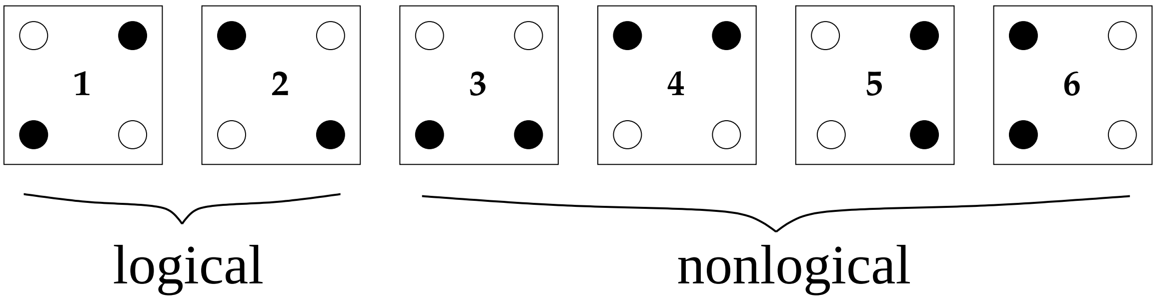



Units: Length $= d$, Energy $= e^2/4\pi\epsilon d$

Assumptions:

- Tunneling energy is very SMALL
- Double occupancy energy is very LARGE

## Six possible configurations:

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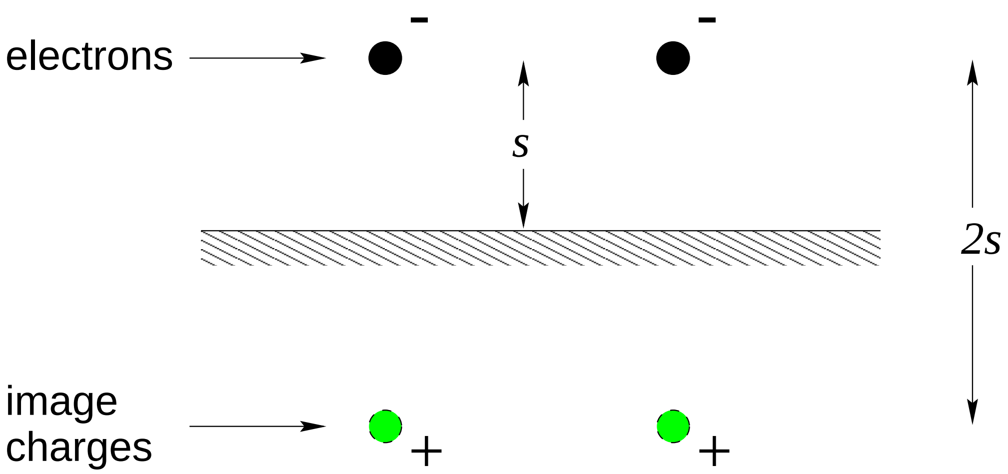
## Charge Neutrality: Ground Plane

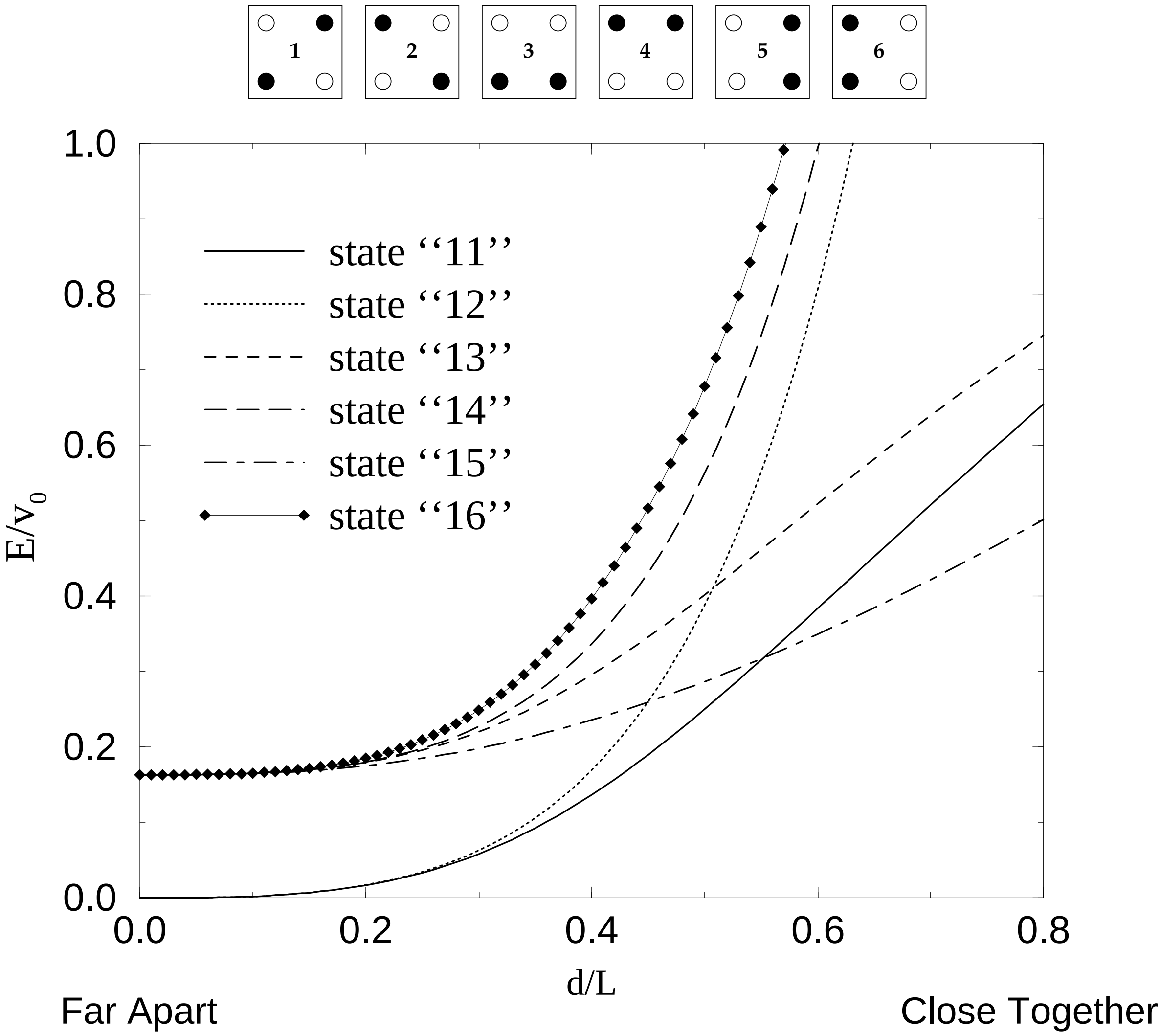



## Positive image charges: screening

$$u(r,s) = u_0(r) - u_0(\sqrt{r^2 + 4s^2})$$

$$u(r,s) \to \frac{e^2}{4\pi\epsilon d} \begin{cases} d/r, & s \to \infty \\ 2ds^2/r^3, & s \to 0 \end{cases}$$

# Energies (Two Cells)




## Three regimes:

- $d/L < d/L_0$: first excited state "12"
- $d/L_0 < d/L < d/L_1$: first excited state "15"
- $d/L_1 < d/L$: $\Delta < 0$, NONLOGIGAL GROUND STATE

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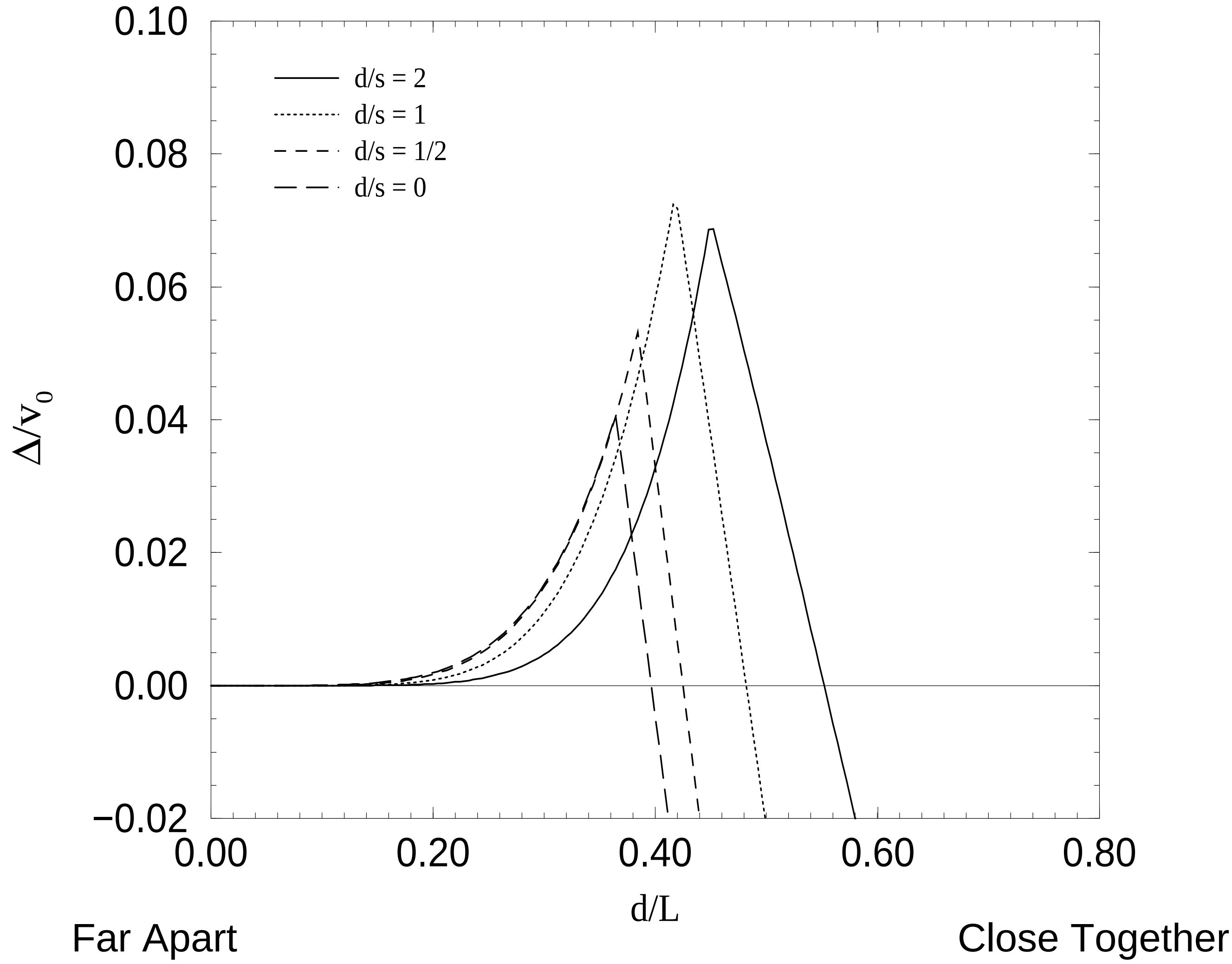
# Energy gap $\Delta$: (Two Cells)




- Increases with $d/L$, then drops quickly
- $\Delta/v_0$ goes negative ... nonlogical ground state
- Maximize $\Delta$ by adjusting $d/L$ and $d/s$

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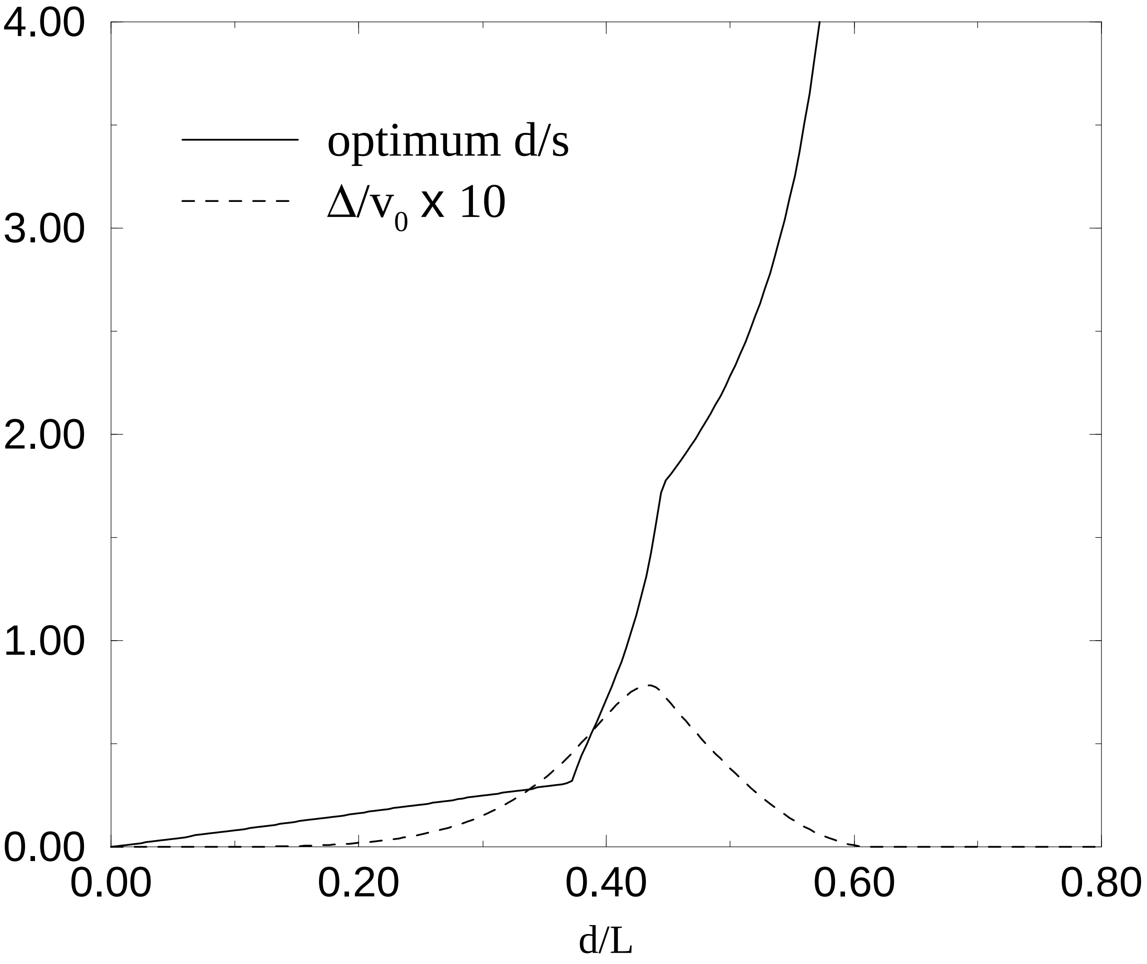
## Optimal ground-plane separation: (Two Cells)




- Maximize $\Delta$ as a function of $d/s$
- Best $\Delta$ at $d/L = 0.432$ and $d/s = 1.313$ (Lent: $d/L = 0.333$)
- No solution with $\Delta \geq 0$ for $d/L > 0.6165$ ...NONLOGICAL GROUND STATE

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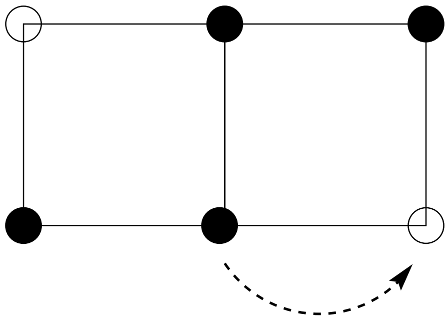
# Nonlogical ground state

- Always get nonlogical ground state as $d/L \to 1$:

  Consider $d/L = 1$

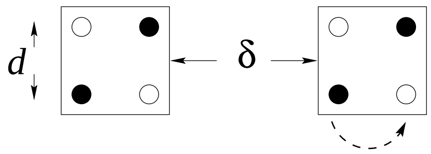


$$\Delta E = [u(d) - u(2d)] < 0$$
$$\text{“11”} \to \text{“15”}$$

- As long as electrons repel $[u(d) > u(2d)]$, get nonlogical state:

$$s = \infty, \ (d/L)_{\max} = 0.3963$$
$$s = 0, \ (d/L)_{\max} = 0.6165$$

- If $u(r) \propto 1/r^n$, then as $n \to \infty$,




$$\lim_{n\to\infty} \delta = d/n$$

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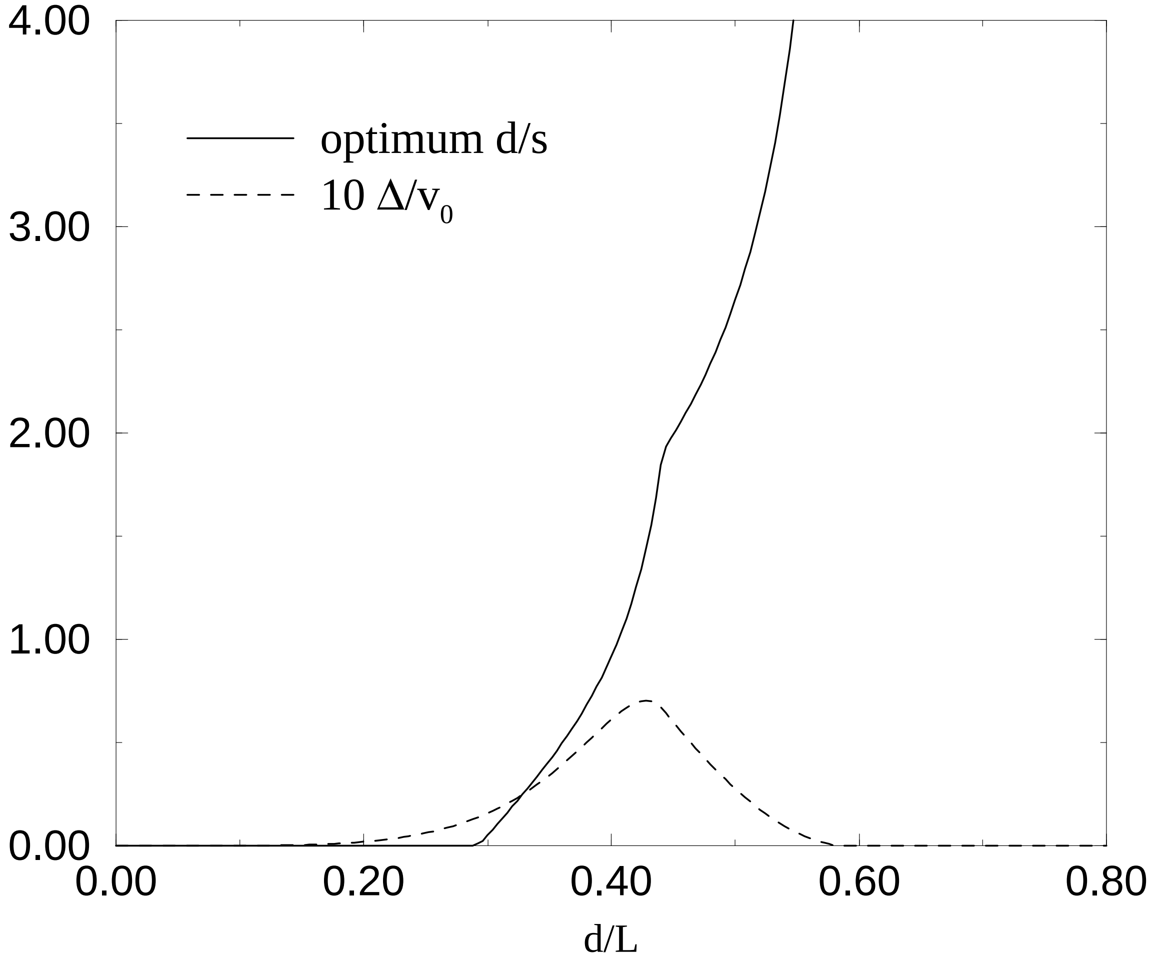
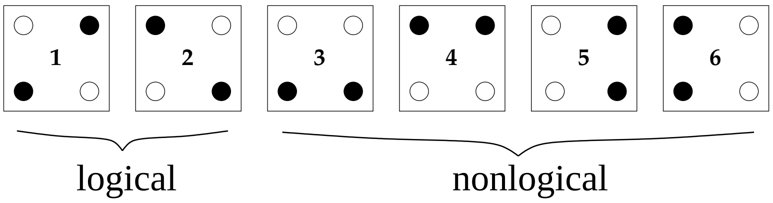
# Infinite Wire

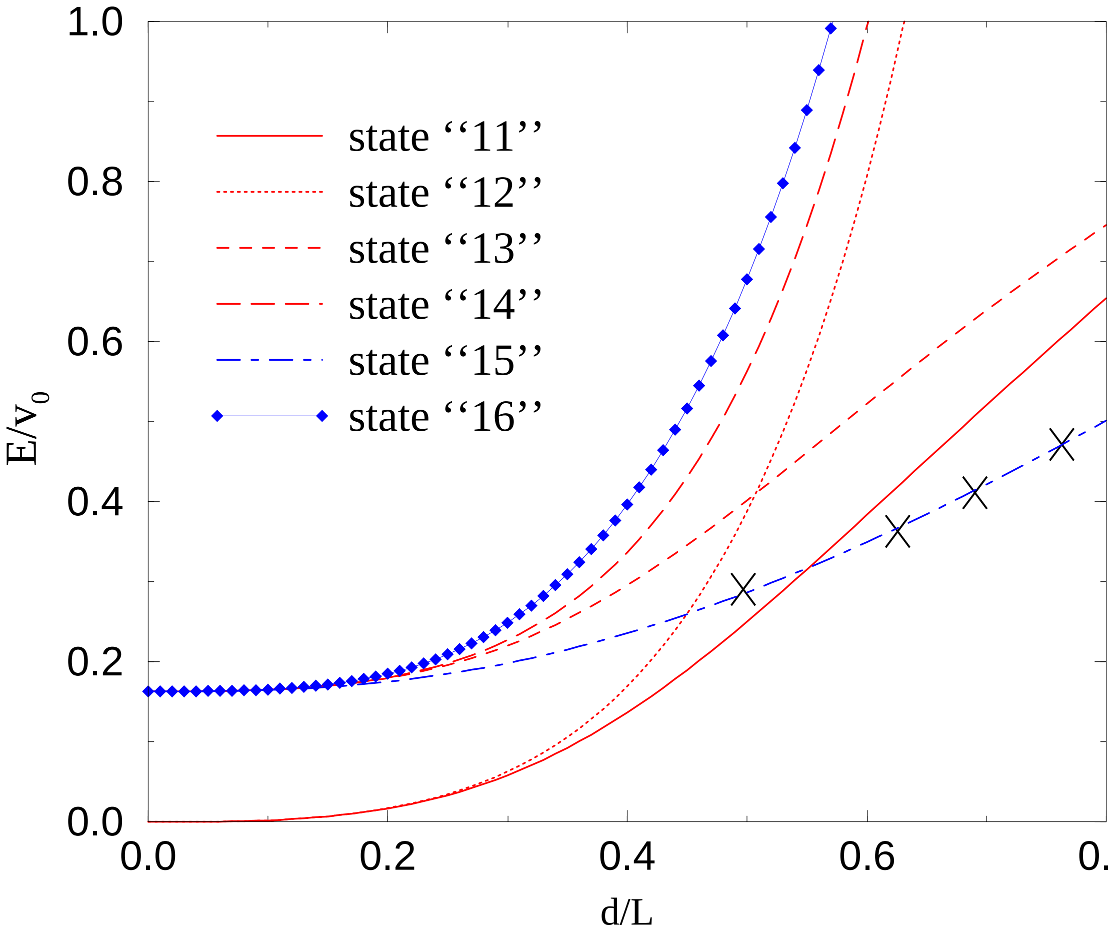



- Similar to two-cell QCA
- Best $\Delta$ at $d/L = 0.428$ and $d/s = 1.44$
- No solution for $d/L > 0.5919$

  . . . NONLOGICAL GROUND STATE

# Eliminating Competing Non-logical States




...increase energy gap $\Delta$

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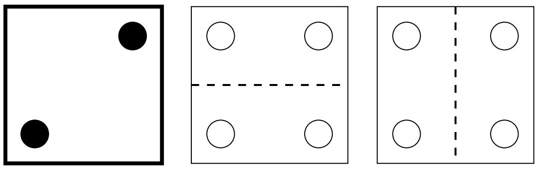
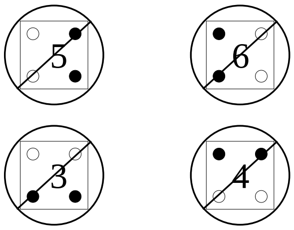
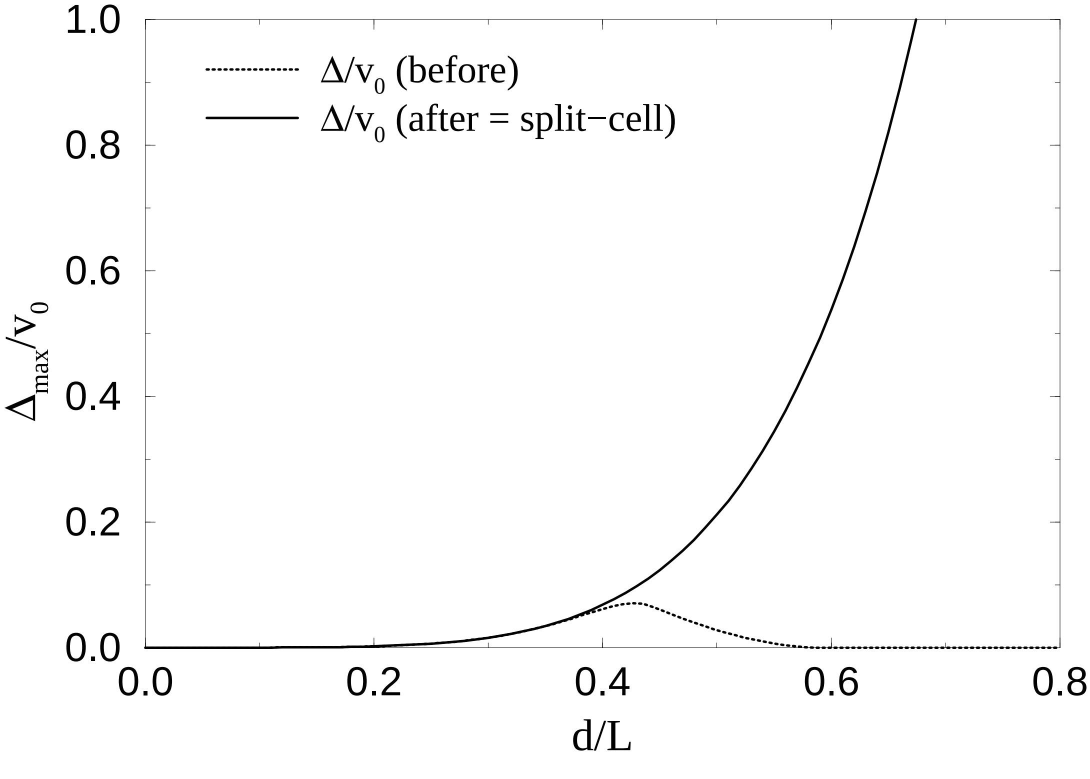
# Split-cell QCA



- Prohibit states 5 and 6 in last cell
- Prohibit states 3 and 4 in middle cells




- Read **output** from **next-to-last cell**




- Substantially larger $\Delta$ for split-cell design
- <u>NO</u> NONLOGICAL GROUND STATE

## Summary:

- Simple Classical Electrostatic Model
  Screened Coulomb interaction (Ground Plane)

$$u(r,s) \to \frac{e^2}{4\pi\epsilon d} \begin{cases} d/r, & s \to \infty \\ 2ds^2/r^3, & s \to 0 \end{cases}$$

- Energy gap: for close enough cells $(d/L = 0.6$ or larger)
  Repulsive $e$ - $e$ interactions always give
  NONLOGICAL GROUND STATE

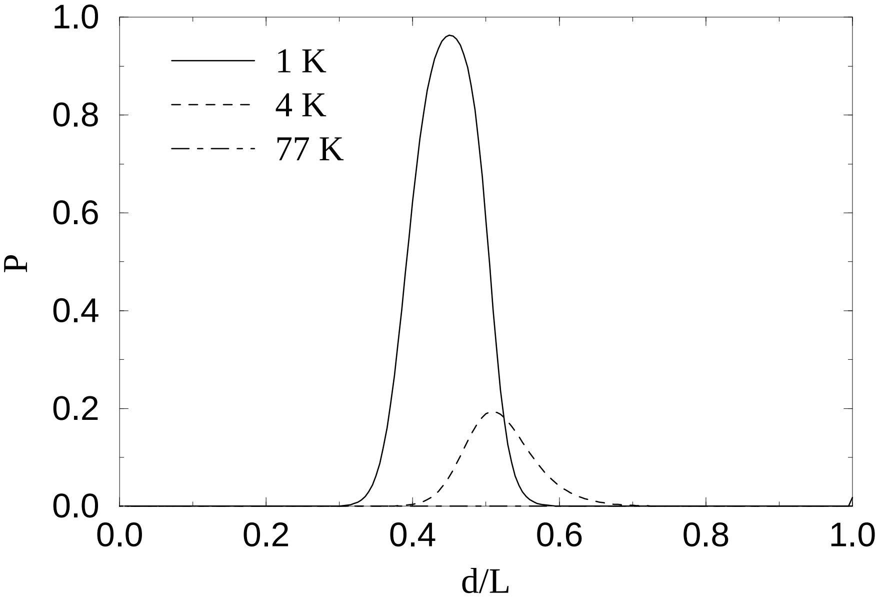
- Optimal ground-plane separation:
  - $\Delta$ can be maximized by varying $L$, $s$

- Alternative Design: Split-cell QCA
  - Eliminating competing states increases $\Delta$
  - Eliminates NONLOGICAL GROUND STATE

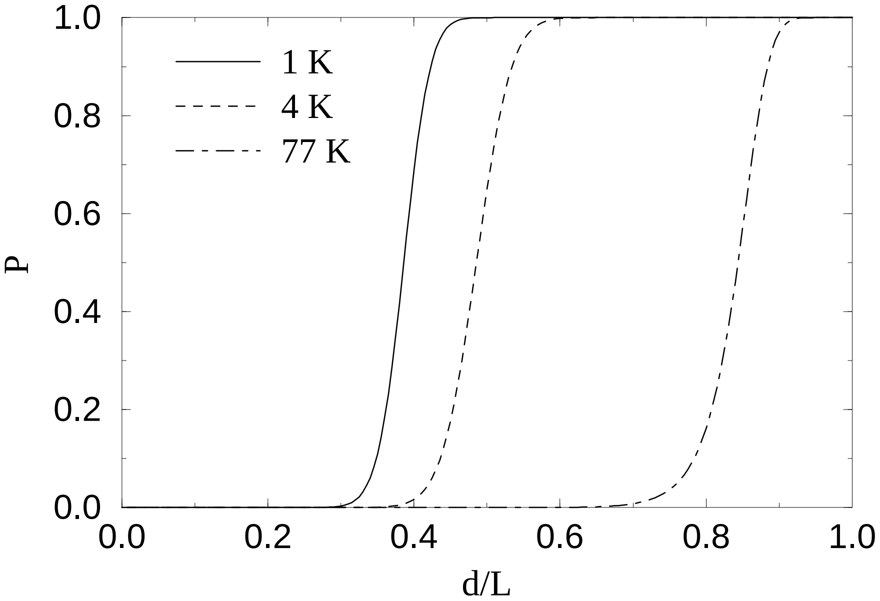
- Finite Temperature
  - Bigger $\Delta \Longrightarrow$ higher operating temperature
  - Performance is improved with split-cell QCA

# Finite-T: 6-cell QCA (Si, $d = 20$ nm)

Polarization: $P =$ Prob("1") - Prob("0")




Split-Cell:

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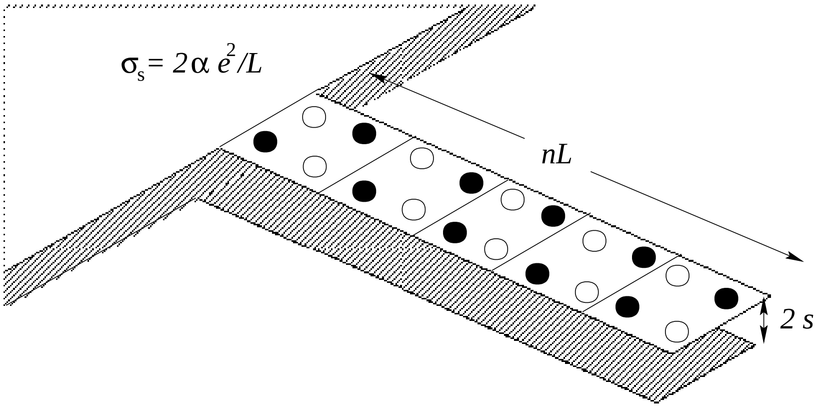
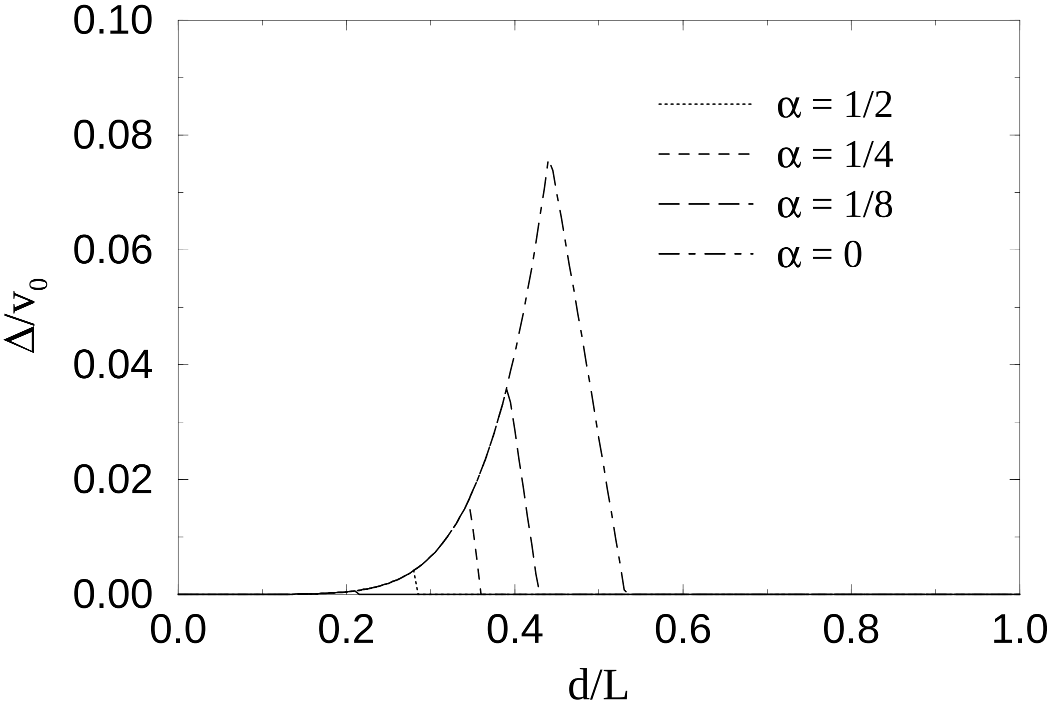
# Nearby QCA:

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