Reweighting Nusim Monte Carlo as a Galactic Plane Line Source

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1 Introduction

A standard procedure for simulating a hypothesized neutrino source is to reweight standard nusim Monte Carlo output to the desired spectrum. Here we consider a procedure to reprocess and reweight Monte Carlo atmospheric neutrino events to simulate a constant line source of neutrinos from the galactic plane.

2 Event Reprocessing

Transforming atmospheric Monte Carlo events into neutrinos from the galactic plane requires two distinct steps. First, the time of the event must be adjusted so that its azimuth ϕ corresponds to an event coming from the plane. Second, the events must be given a user-defined weight, so that they are distributed isotropically in galactic longitude.

2.1 Event Time

During a day at the South Pole, a given point on the galactic plane rotates around the pole at a fixed zenith angle. So, given an event with zenith angle θ and azimuth ϕ, one can determine the time of day to assign that event so that it will land on the galactic plane (or alternatively, so that the galactic plane will land on the event!).

Given a declination δ in the range spanned by the galactic plane, there are in general two points of right ascension α_1 and α_2 such that (α_1, δ) and (α_2, δ) lie on the galactic equator (see figure 1). So for an event with declination δ, we can find one of these points via the equation of the plane in equatorial coordinates:

$$\alpha = \arcsin(\tan \delta_{NGP} \ \frac{\sin \delta}{\sqrt{1 - \sin^2 \delta}}) + \alpha_0 \ . \tag{1}$$

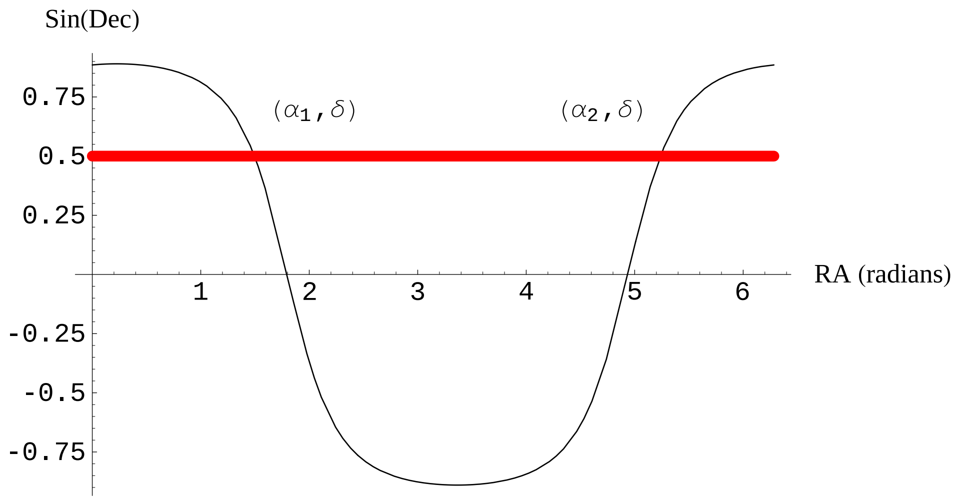

Figure 1: Two Points on the Galactic Plane at a Given Declination

The constant δ_{NGP} is the declination of the North Galactic Pole, and α_0 is the RA (in radians) where the galactic plane crosses the celestial equator. Equation 1 can be quickly derived from the general coordinate transformations [1] by setting $b = 0$. However, the arcsin() only results in half of the plane, and thus only one of the possible points; the other one can be calculated by reflecting across the symmetry axis at $\alpha_0 - \frac{\pi}{2}$.

Next, a random day of the year is chosen, and using one of the two target RA points (also chosen at random), the time of day necessary to land the event azimuth ϕ on the RA α is calculated:

$$t(sec) = (\frac{\alpha(degrees) - 90 + \phi}{15} - 6.5988098 - 0.0657098244 \cdot day) \cdot 3600/1.00273791 \ . \qquad (2)$$

Specifically, equation 2 gives the target time for a given day in the year 2000. The coefficients must be adjusted slightly for each year [2]. Corrections due to nutation and the difference between UT1 and UTC are not included, but both of these are less than a second.

Once the event is assigned the day and time as described above, it's now coming from the direction of the galactic plane.

2.2 Galactic Weight

Next, the events must be reweighted so that they are isotropically distributed in galactic longitude l. At the time of generation (ignoring any earth or atmospheric effects, since these are not changed by this procedure), Monte Carlo events are isotropically distributed in $\sin\delta$; that is, $dn/d\sin\delta = C$, for a cumulative distribution function $n(\sin\delta)$. The constant C is not important for this argument, as we deal with absolute flux normalization in the next section. We are looking for a weighting function $w(\sin\delta)$ so that

$$\frac{dn}{dl}\; w(\sin\delta) = C \; . \tag{3}$$

Now, since

$$\frac{dn}{d\sin\delta} = \frac{dn}{dl}\frac{dl}{d\sin\delta} = C \; , \tag{4}$$

we see that the weight is just the Jacobian of the one-dimensional transformation from declination to galactic longitude; that is,

$$w(\sin\delta) = \frac{dl}{d\sin\delta} \; . \tag{5}$$

The equation of transformation from equatorial to galactic coordinates for the galactic plane ($b = 0$) is the following:

$$l = \arcsin(\frac{\sin\delta}{\cos\delta_{NGP}}) + l_0 \tag{6}$$

where l_0 and δ_{NGP} are constants. Thus the *galactic weight* for each event is:

$$w_g(\sin\delta) = \frac{1}{\sqrt{1 - \frac{\sin^2\delta}{\cos^2\delta_{NGP}}}} \; . \tag{7}$$

Figure 2 shows a plot of the galactic weight as a function of $\sin(\delta)$. Around the celestial equator ($\sin(\delta) = 0$), the weight is fairly uniform, but as the galactic plane reaches its extremes of declination, the weight blows up. This has the side effect of magnifying statistical fluctuations in the event sample at these declinations, but these can be controlled by using a larger number of events and/or clamping the weight to a maximum value.

3 Flux Normalization

Even once the events are retimed and individually reweighted, they still must be weighted again to obtain a meaningful flux normalization. This is similar to a normal E^{γ} normalization, but not quite, because our flux from the galactic plane is a linear flux per radian, not a point source, or a diffuse flux per steradian.

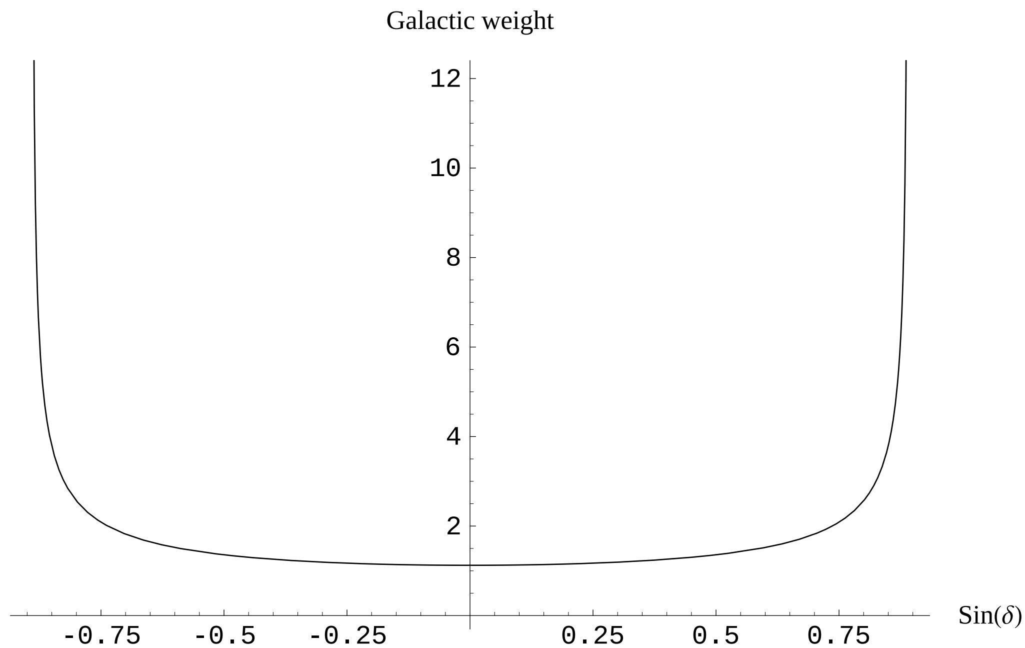

Figure 2: Galactic Weight as a Function of Sin(Declination)

The standard weight for a diffuse E^γ flux Φ is the following [3]:

$$w_i = intwght \cdot r1 \cdot r1^\gamma \cdot r14 \cdot 1e10 \cdot 86400 \cdot d \cdot \Omega \cdot \Phi \cdot \frac{16.12}{2\ N} \tag{8}$$

where $intwght$, $r1$, and $r14$ are standard energy- and interaction-dependent nusim reweighting parameters, d is the livetime in days, N is the total number of ν or $\overline{\nu}$ events, and Ω is the solid angle of generation.

The goal is to find an effective diffuse flux Φ_{eff} which, when combined with the galactic weight discussed in the previous section, corresponds to a given linear flux Φ_{gal} from the galactic plane:

$$w_i = intwght \cdot r1 \cdot r1^\gamma \cdot r14 \cdot 1e10 \cdot 86400 \cdot d \cdot \Omega \cdot \Phi_{eff} \cdot w_g \cdot \frac{16.12}{2\ N}\ . \tag{9}$$

In a small band of declination $\Delta \sin \delta$, the galactic flux from a section (two sections, actually) of longitude Δl should be the same as the galactic-reweighted flux from our effective Φ_{eff} integrated over 2π of azimuth. That is,

$$\Phi_{gal}\ \Delta l = 2\pi\ w_g\ \Phi_{eff}\ \Delta \sin \delta\ . \tag{10}$$

Summing over the region we're interested in (the galactic plane below the horizon) and taking the integral limit, we have

$$\int_{l_{min}}^{l_{max}} \Phi_{gal}\ dl = 2\pi\ \int_{0}^{\sin\delta_{max}} w_g\ \Phi_{eff}\ d\sin\delta\ . \tag{11}$$

Solving for Φ_{eff}, we find:

$$\Phi_{eff} = \frac{\Phi_{gal}}{2\pi}\ (\int_{l_{min}}^{l_{max}} dl)\ (\int_{0}^{\sin\delta_{max}} w_g\ d\sin\delta)^{-1}\ . \tag{12}$$

But since $w_g = dl/d\sin\delta$, the right-hand-side integral over the declination becomes:

$$\int_{0}^{\sin\delta_{max}} \frac{dl}{d\sin\delta}\ d\sin\delta = l(\sin\delta_{max}) - l(\sin\delta = 0) = \frac{l_{vis}}{2} \tag{13}$$

where l_{vis} denotes the total longitude visible below the horizon. The integral only results in half of l_{vis} since the plane crosses this declination region twice. Furthermore, the other integral is just the total l_{vis}:

$$\int_{l_{min}}^{l_{max}} dl = l_{vis}\ . \tag{14}$$

Using these results, the expression for the effective flux simplifies considerably:

$$\begin{aligned}\Phi_{eff} &= \frac{\Phi_{gal}}{2\pi}\ \frac{l_{vis}}{l_{vis}/2} \\ &= \frac{\Phi_{gal}}{\pi}\ .\end{aligned} \tag{15}$$

We now have Φ_{eff}, a flux per steradian, for use in the reweighting expression. The remaining task is to consider the generation solid angle Ω, since not all of the generated events are used in the signal simulation (some fall outside of the declination range of the galactic plane).

If we had generated events only in the range of the galactic plane Ω_{gal}, for the same absolute flux we would expect a scaled number of events $N \cdot \frac{\Omega_{gal}}{\Omega_{gen}}$. Thus the relevant product in the reweighting expression becomes:

$$\frac{\Omega_{gal}\ \Phi_{eff}}{N_{scaled}} = \frac{\Omega_{gal}\ \Phi_{eff}}{N}\ \frac{\Omega_{gen}}{\Omega_{gal}} \tag{16}$$

$$= \frac{\Omega_{gen}\ \Phi_{eff}}{N}\ . \tag{17}$$

So the galactic solid angle cancels out, and the generation solid angle remains the same as in equation 8.

Therefore, we have for the final normalized reweighting expression for a linear galactic flux Φ_{gal}, including the galactic weight for each event w_g:

$$w_i = intwght \cdot r1 \cdot r1^{\gamma} \cdot r14 \cdot 1e10 \cdot 86400 \cdot d \cdot \Omega_{gen} \cdot \frac{\Phi_{gal}}{\pi} \cdot w_g(\sin\delta) \cdot \frac{16.12}{2\ N}\ . \tag{18}$$

References

[1] Leinert, Ch. *et al.*, The 1997 reference of diffuse night sky brightness. Astron. Astrophys. Suppl. Ser. **127**, 5-6 (1998).

[2] *The Astronomical Almanac*. U.S. Goverment Printing Office, Washington, D.C. (2000-2003).

[3] See http://www.amanda.wisc.edu/simulation/weight.html.