

# Testing Lorentz Invariance using Atmospheric Neutrinos and AMANDA-II

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## 1 Abstract

2 Several phenomenological models of physics beyond the Standard Model predict flavor  
3 mixing in the neutrino sector in addition to conventional mass-induced oscillations. In  
4 particular, violation of Lorentz invariance (VLI) results in neutrino oscillation effects  
5 parametrized by the maximal attainable velocity difference  $\delta c/c$ . We report on a study of  
6 the sensitivity of the AMANDA-II detector to such effects using distortions in the spec-  
7 trum of high-energy atmospheric neutrinos. For maximal mixing and six years of simulated  
8 data, the preliminary sensitivity of AMANDA-II to VLI of this type is  $\delta c/c < 2.1 \times 10^{-27}$   
9 at the 90% confidence level.

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## 10 1. Introduction

11 Flavor oscillations in the neutrino sector provide an interesting method to test  
12 phenomenological models of physics beyond the Standard Model. While mass-  
13 induced oscillations of atmospheric neutrinos are on firm experimental footing [1–3],  
14 subdominant effects may yet be present. In particular, violation of Lorentz invari-  
15 ance (VLI) can result in oscillations at high energies and distort the atmospheric  
16 neutrino spectrum.

17 The AMANDA-II detector, a subdetector of the IceCube experiment, is an array  
18 of 677 optical modules buried in the ice at the geographic South Pole which detects  
19 the Čerenkov radiation from charged particles produced in neutrino interactions  
20 with matter [4]. In particular, muons produced in charged-current  $\nu_\mu$  and  $\bar{\nu}_\mu$  inter-  
21 actions deposit light in the detector with a track-like topology, allowing us to use  
22 directional reconstruction to reject the large background of down-going atmospheric  
23 muon events. After suitable quality selection criteria are applied, AMANDA-II ac-  
24 cumulates atmospheric neutrino candidates above 50 GeV at a rate of  $\approx 4$  per day  
25 [5]. While conventional oscillations are suppressed at these energies, VLI effects can  
26 be detected or constrained by their influence on the zenith angle distribution and  
27 energy-correlated observables.

## 28 2. Phenomenology

29 Various new physics scenarios can result in neutrino flavor mixing beyond con-  
30 ventional oscillations. We focus here on oscillations induced by differing maximally  
31 attainable velocities (MAVs) in the neutrino sector. MAV eigenstates can be dis-  
32 tinct from flavor eigenstates, resulting in oscillations characterized by the MAV  
33 difference  $\delta c/c = (c_1 - c_2)/c$ .

34 Conventional and VLI oscillations can be combined in a two-family scenario, with  
 35 the following survival muon neutrino survival probability as a function of energy  $E$   
 36 and baseline  $L$  (in energy units) [6–8]:

$$P_{\nu_\mu \rightarrow \nu_\mu} = 1 - \sin^2 2\Theta \sin^2 \left( \frac{\Delta m^2 L}{4E} \mathcal{R} \right), \quad (1)$$

37 where

$$\sin^2 2\Theta = \frac{1}{\mathcal{R}^2} (\sin^2 2\theta + R^2 \sin^2 2\xi + 2R \sin 2\theta \sin 2\xi \cos \eta), \quad (2)$$

$$\mathcal{R} = \sqrt{1 + R^2 + 2R(\cos 2\theta \cos 2\xi + \sin 2\theta \sin 2\xi \cos \eta)}, \quad (3)$$

and

$$R = \frac{\delta c}{c} \frac{E}{2} \frac{4E}{\Delta m^2}. \quad (4)$$

38 Standard oscillations are characterized by the mass-squared difference  $\Delta m^2$  and  
 39 mixing angle  $\theta$ , while the VLI oscillation parameters include the velocity difference  
 40  $\delta c/c$ , the mixing angle  $\xi$ , and the phase  $\eta$ . If we take both conventional and VLI  
 41 mixing to be maximal ( $\theta = \xi = \pi/4$ ) and set  $\cos \eta = 1$ , this reduces to the following:

$$P_{\nu_\mu \rightarrow \nu_\mu}(\text{maximal}) = 1 - \sin^2 \left( \frac{\Delta m^2 L}{4E} + \frac{\delta c}{c} \frac{LE}{2} \right). \quad (5)$$

42 Note the different energy dependence of the two effects. For atmospheric neu-  
 43 trinos, the zenith angle functions as a surrogate for the baseline  $L$ , allowing path  
 44 lengths up to the diameter of the Earth. Figure 1 shows the survival probability as  
 45 a function of neutrino energy and zenith angle for the maximal case, as in equation  
 46 5.

### 47 3. Analysis Methodology

48 First, to obtain a clean sample of atmospheric neutrinos, we must separate these  
 49 from the large background of atmospheric muons. Selecting events with a recon-  
 50 structed zenith angle below the horizon allows rejection of many such events, but we  
 51 must generally apply further quality criteria to eliminate mis-reconstructed muons.  
 52 For this study, we have used the selection criteria from the 2000-03 AMANDA-II  
 53 point source search [5] and examine only zenith angles  $> 100^\circ$ .

54 Next, our goal is to measure or constrain the energy-dependent angular distor-  
 55 tions caused by VLI effects. While AMANDA-II has an angular resolution of a few  
 56 degrees [9], reconstruction of the neutrino energy is more difficult and fundamen-  
 57 tally limited by the stochastic losses of the muon. Instead, we use a well-simulated  
 58 energy-correlated observable, the number of triggered optical modules ( $N_{ch}$ ).

59 Now, to determine values of the parameters  $\theta_i$  of our hypothesis (in the simplest  
 60 one-dimensional case, just  $\delta c/c$ ) that are allowed or excluded at some confidence  
 61 level, we follow the likelihood prescription described by Feldman and Cousins [10]:

- 62
- 63 – For each point in the parameter space  $\theta_i$ , we sample many times from the parent
  - 64 Monte Carlo distributions of the observable(s) (MC “experiments”).
  - 65
  - 66 – For each MC experiment, we calculate the log likelihood ratio

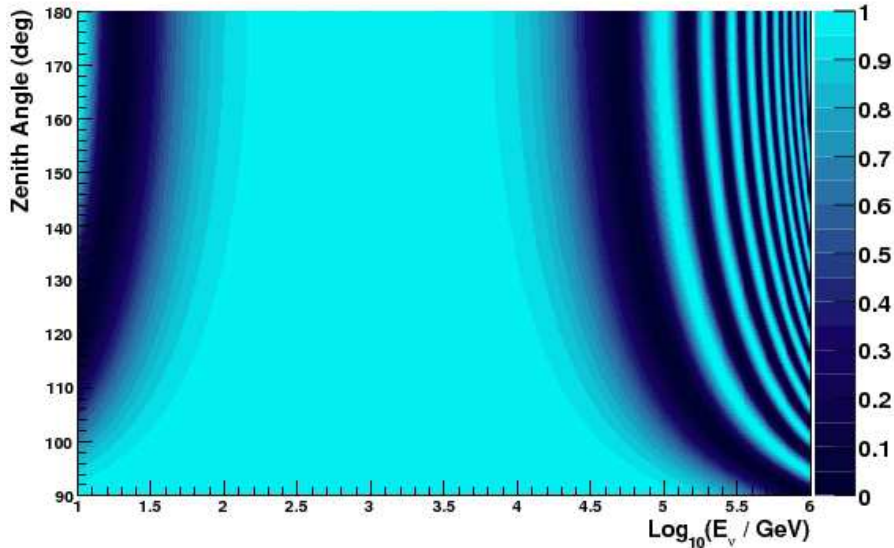


Fig. 1. Atmospheric  $\nu_\mu$  survival probability as function of neutrino energy and zenith angle. Conventional oscillations are present at low energies, while high-energy oscillations are due to VLI (maximal mixing,  $\delta c/c = 10^{-27}$ ).

$$\Delta\mathcal{L} = -2 \ln L_i + 2 \ln L_{i,best} , \quad (6)$$

67 where  $L_i$  is the Poisson probability that the MC experiment is derived from the  
 68 parent distribution at  $\theta_i$  (other likelihood formulations are possible).

69  
 70 – For each point  $\theta_i$ , we find the value  $\Delta\mathcal{L}_{crit}$  at which, say, 90% of MC experiments  
 71 have a lower  $\Delta\mathcal{L}$ .

72  
 73 – Finally, we compare the  $\Delta\mathcal{L}$  of the data (or in our case, a simulated data set gen-  
 74 erated under the null hypothesis) with the critical surface  $\Delta\mathcal{L}_{crit}$ , and regions of  
 75 the parameter space at which  $\Delta\mathcal{L} > \Delta\mathcal{L}_{crit}$  are excluded at that confidence level.  
 76 For a one-dimensional parameter space, this can likely be interpreted an upper  
 77 limit, and one can calculate a median sensitivity by iterating over a number of  
 78 simulated data sets.

79  
 80 As noted in [10], the likelihood formulation has a number of desirable features  
 81 compared to a standard  $\chi^2$  approach, the most significant being proper coverage.

#### 82 4. Sensitivity of AMANDA-II

83 We have performed a Monte Carlo study using six years of simulated AMANDA-  
 84 II data: an integrated exposure of 1200 days, approximately 5100 events below the  
 85 horizon under the null hypothesis (conventional oscillations only). For this initial  
 86 study, we have tested only the  $N_{ch}$  distribution across a one-dimensional param-  
 87 eter space, varying the VLI strength  $\delta c/c$ . To anticipate the impact of the inclu-  
 88 sion of systematic errors in the future, we have left free the normalization of the

89 atmospheric neutrino flux (*i.e.* treating it as a nuisance parameter). We have not  
 90 included the zenith angle distribution in this analysis, as we have not yet accounted  
 91 for systematic uncertainties in the shape of the spectrum. The curves of  $\Delta\mathcal{L}_{crit}$  for  
 92 the 90%, 95%, and 99% confidence levels are shown in Figure 2, along with the  
 93 likelihood ratio for a single simulated data set.

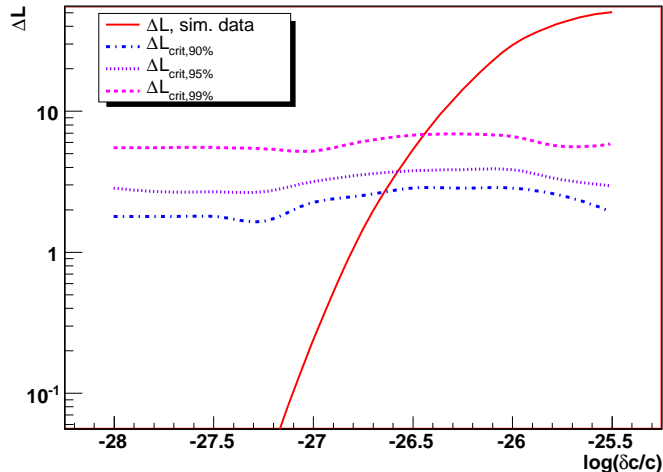


Fig. 2. Likelihood ratio for VLI effects using the shape of the  $N_{ch}$  distribution, for values of the parameter  $\delta c/c$ . The critical curves for various confidence levels are shown, along with  $\Delta\mathcal{L}$  for a simulated six-year data set. Values of  $\delta c/c$  to the right of the point of intersection with the critical curve are excluded.

94 Assuming maximal mixing ( $\sin 2\xi = 1$ ) and phase  $\cos \eta = 1$ , we find a median  
 95 sensitivity of  $\delta c/c < 2.1 \times 10^{-27}$  at the 90% confidence level. Existing experimental  
 96 limits include the MACRO result of  $\delta c/c < 2.5 \times 10^{-26}$  [11] and the limit by  
 97 González-García and Maltoni using the Super-Kamiokande + K2K data,  $\delta c/c <$   
 98  $2.0 \times 10^{-27}$  [8].

## 99 5. Conclusions and Outlook

100 Using its large sample of atmospheric neutrinos, AMANDA-II is capable of de-  
 101 tecting or constraining high-energy new physics effects in the neutrino sector. The  
 102 Monte Carlo study presented here indicates a sensitivity to VLI effects competi-  
 103 tive with existing limits, and a number of improvements (such as testing multiple  
 104 observables) and optimizations (*e.g.* event selection criteria, and the binning of  
 105 the observables) are forthcoming. We anticipate applying this analysis in the near  
 106 future to the AMANDA-II data collected during 2000-2005.

107 Furthermore, the same methodology can also be applied to constrain other physics  
 108 beyond the Standard Model, such as violations of the equivalence principle [13] or  
 109 quantum decoherence resulting from interactions of neutrinos with the background  
 110 space-time foam [14–16].

111 The next-generation IceCube detector, with an instrumented volume of  $1 \text{ km}^3$ ,  
 112 will allow unprecedented sensitivity to these same effects. In 10 years of operation,  
 113 IceCube will collect a sample of over 700 thousand atmospheric neutrinos and will

114 be sensitive at the 90% confidence level to VLI effects at the level of  $\delta c/c < 2.0 \times$   
115  $10^{-28}$  [12]. This high-statistics sample will also provide an opportunity to test other  
116 phenomenological models of physics beyond the Standard Model.

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