**Ray Showers**

**NOTICE THAT**

\[
\frac{\frac{3}{2} \text{pair}}{\frac{3}{2} \text{brems}} = \frac{\text{brems}}{\text{pair}} = \frac{\frac{2}{3} \ln \frac{2E}{m_e}}{4 \left( \ln \frac{123}{248} + \frac{1}{8} \right)}
\]

\[= 1 \text{ for } E = 1 \text{ GeV } Z = 7-8 \text{ (air)}\]

We can treat a high energy shower as if radiation and pair production occur after the same distance.

After \((n-1)\) interactions the energy of a particle (e or \(\gamma\), roughly \(\frac{1}{3} E_{e^+X} \text{ and } \frac{1}{2} E_\gamma\)) has degraded to \(E_0/2^n \approx E_{\text{critical}} \approx m_e\). The shower reaches its "maximum". (\(\rightarrow \text{ctd}\))
\[ x = \eta \frac{E_0}{E_c} \ln z \]

\[ n_{\text{max}} = \frac{E_0}{E_c} \]

\[ \int dx E \frac{dN}{dE} dx = \int \frac{2^n}{\ln 2} \frac{d\ln y}{y} \]

\[ \frac{2^n}{\ln 2} \frac{d\ln z}{z} = \frac{d\ln y}{y} \]

\[ dn = \frac{1}{\ln 2} \frac{dy}{y} \]

\[ n_{\text{max}} = \frac{1}{\ln 2} \frac{E_0}{E_c} \]

\[ \frac{dN}{dE} = \left( \frac{\frac{E_0}{\ln 2 E_c}}{} \right) \frac{1}{E} \]
It is absorbed by the atmosphere, $e^\pm$ ionizes atoms and $\gamma$'s are lost by photoelectric absorption.

**Properties of the shower**

$R$ is the distance after which the $\gamma$ has lost $1/2$ of its primary energy $E_0$

$$e^{-\frac{R}{R_0}} = \frac{1}{2} \quad \rightarrow \quad R = R_0 \ln 2$$

At shower max

$$\frac{E_0}{2^{n_{\text{max}}}} = E_\gamma \quad \rightarrow \quad n_{\text{max}} = \frac{\ln (E_\gamma/E_0)}{\ln 2}$$

number of particles at max ~ energy $E_\gamma$

$$n = 2^{(n_{\text{max}} + 1)} = 2 \left(\frac{E_\gamma}{E_0}\right)^{n_{\text{max}}}$$

exponential growth to $n_{\text{max}}$ radiation length followed by rapid attenuation

spectrum

$$n(E,R) = 2^R$$

$$\int_0^{R_{\text{max}}} 2^R \, dR = \frac{E_0}{E} \ln 2$$

$$R_{\text{max}} = \frac{\ln (E_0/E)}{\ln 2}$$

$E = 100 \text{ MeV}$

$E = 1 \text{ MeV}$

$$E = 10^5$$

$$E_{\gamma} = 10^5$$

$$n_{\text{max}} = \ln (10^5) = 15.5$$

$$n = 2 \times 10^5$$
Fig. 4.6. The total number of particles $N$ in a shower initiated by an electron of energy $E_0$, as a function of depth $n$, measured in radiation lengths; $E_c$ is the critical energy of the material. (From Leighton, 1959, p. 693, after Rossi & Greisen, 1941.)
Cascade Equations

One of the most important techniques in high energy astrophysics. E.g.
- the problem of a particle beam (from pulsars) propagating through companion atmosphere
- particles propagating outward from the sun's interior
- propagation of particles through ISM
- propagation of CR through atmosphere

We will discuss the last problem but the techniques are the same for all other problems.

\[
\frac{dN_i(E,x)}{dx} = \Theta \frac{1}{\lambda_i} N_i(E,x) \Theta \frac{1}{d_i} N_i(E,x)
\]

is produced in interaction \(i\) on \(N_j\,O\)
Symbols in the cascade equation

- \( N_i(E, x) \): number of particles \( i \) at a depth \( x \) (in \( g/cm^2 \)) in the atmosphere of energy \( E \)

- \( \lambda_i \): average depth at which the particles interact (interaction or radiation length, in \( g/cm^2 \)) (see 83)

- \( d_i \): average decay length if \( X \) is unstable \((d_i = \rho \lambda_i)\)

- \( F_{ji} \): dimensionless cross section for a particle \( j \) of energy \( E_j \) to produce a particle of type \( i \) and energy \( E_i \) in an interaction with an air nucleus

\[
\alpha = 0
\]

\[
\alpha_v \quad \frac{d}{dx} \quad \frac{d}{dx}
\]

\[
h = 0
\]

- \( h \): height
- \( \theta \): zenith angle

\[
\frac{\Delta N_i}{N_i} = \frac{\Delta d}{\gamma \lambda_{Ti}}
\]

\[
\frac{\Delta x}{\rho \gamma \lambda_{Ti}}
\]

\[
\ln \frac{d}{d_i} = E_i \frac{dN_i}{dE_i} \left( \frac{E_i}{E_j} \right)
\]

\[
\frac{1}{\rho} \frac{\alpha_v}{d} \quad \frac{d}{dx}
\]
- total cross section of particle $j$ on air is $\sigma_j$

- $d\sigma/d\alpha$ is the inclusive cross section for $j$ to produce $i$ with a fraction $\alpha = E_i/E_j$ of its original momentum measured by accelerators

notice ( $E_{ij} \gg m_{ij}$ )

$\rightarrow \quad 0 \leq \alpha = \frac{E_i}{E_j} \leq 1$

$\rightarrow \quad \int_0^1 dx_\alpha \left[ \frac{1}{\sigma_j} \frac{d\sigma_i(x)}{dx_\alpha} \right] = \langle m_i \rangle$

$\rightarrow \quad \int_0^1 dx_\alpha x_\alpha \left[ \frac{1}{\sigma_j} \frac{d\sigma_i(x)}{dx_\alpha} \right] = \langle x_i \rangle$

$\langle x_i \rangle$, $\langle x_i \rangle$ is the average number (multiplicity) and the average (relative) momentum of particles $i$ produced in a particle-air collision.
Cascade Equations in Approximation A

Assumptions:

1. Scaling: \( F_{ji} (E_i, E_j) \) depends on \( x_L = E_i / E_j \) only.

2. \( \lambda \) independent of \( E_j \), i.e. cross section vary weakly with energy.

The cascade eq of (110) can now be written as:

\[
\frac{dN_i(E,x)}{dx} = - \left( \frac{1}{x_i} + \frac{1}{d_i} \right) N_i(E,x)
\]

\[
+ \frac{1}{x_j} \int_0^x \frac{d\alpha_L}{\alpha_L^2} \left[ \frac{x_L}{d_j} \frac{d\sigma}{dx_L} (x_L) \right] N_j \left( \frac{E}{x_L} \right)
\]

Equation:

\[ x_L = \frac{E_i}{E_j} = \frac{E}{x_L} \]

\[
\frac{dE_j}{E_i} = d\left( \frac{1}{x_L} \right) = - \frac{dx_L}{x_L^2}
\]

\[ E_j = \frac{E_i}{x_L} \]

\[ \frac{E}{x_L} \]
Nucleons in the atmospheric cascades (or elementary solutions of the cascade eq.)

\[ N(E,x) \text{ flux of nucleons} \]

\[
\frac{dN(E,x)}{dx} = -\frac{1}{\lambda_N} N(E,x)
\]

\[
\left( + \int \frac{dx_L}{x_L^2} \frac{N(E/x_L)}{\lambda_N} \left[ \frac{x_L}{\sigma_N} \frac{d\sigma_{NN}(x_L)}{dx_L} \right] \right)
\]

\[ \lambda_N : \text{ interaction length of nucleons in air.} \]
(defined as radiation length, see (78))

\[
\lambda_N = \frac{\rho(\text{air})}{\rho(\text{nucleons})} \frac{1}{\sigma_N^{\text{air}}} = 80 \ \text{g/cm}^2
\]

\[ 14.4 \text{ averaged over N}_2O \]

\[
\lambda_N = \frac{A m_N}{\sigma_N^{\text{air}}}
\]

\[ \sigma_N^{\text{air}} = (14.4)^{2/3} \sigma_{NN} \]

50 mb

NN total cross section
The nucleon cascade eq. (114) has to be solved with the boundary condition at the top of the atmosphere ($x = 0$)

- $N(E, 0) = A \delta(E - \frac{E_0}{A})$ i.e. nucleus($\text{with A nucleons}$)
  - depends on energy
  - region fitted (composition)
- $N(E, 0) = \frac{dN}{dE} = 1.8 \times E^{-2.7}$ (in $E \lesssim 10^3$ TeV)
  - (nucleons in cm$^2$ sr GeV/A)
  - i.e. cosmic ray flux

**Elementary Solution**

$$N(E, x) = G(E) \cdot g(x)$$

Substitute in (114)

$$G \cdot \frac{dg}{dx} = - \frac{G \rho q}{\lambda_N} + q \int_0^1 \frac{dx_L}{x_L^2} \frac{G(E|x_L)}{\lambda_N} \left[ \frac{1}{x_N} \frac{d\sigma_{NN}}{dx_N} \right]$$

$$\frac{dg}{dx} = - \frac{1}{\Lambda} ; \quad \frac{1}{\Lambda} = \frac{1}{\lambda_N} - \frac{1}{G(E)} \int_0^1 \frac{dx_L}{x_L^2}$$

$$g(x) = g(0) e^{-\frac{x}{\Lambda}}$$
Nucleons (ctd)

use previous solution with the boundary condition

\[ G(E) \sim E^{-\gamma+1} \quad (\gamma = 1.7 \text{ for CR}) \]

\[
N(E, x) = g(0) e^{-\frac{\lambda x}{\lambda N}} E^{-\gamma+1}
\]

\[
\frac{1}{\lambda N} = \frac{1}{\lambda N} \left[ 1 - \int_0^{xL} \frac{d\lambda L}{\lambda L} \frac{G(E/\lambda L)}{G(E)} \left[ \frac{\lambda L}{\lambda N} \frac{d\rho_{NN}}{d\lambda L} \right] \right]
\]

\[
\frac{1}{\lambda N} = \frac{1}{\lambda N} \left[ 1 - \int_0^{xL} xL \lambda L^{-\gamma-1} \left[ \frac{\lambda L}{\lambda N} \frac{d\rho_{NN}}{d\lambda L} \right] \right]
\]

Discussion:

Above equation describe the flux of nucleons in the atmosphere initiated by CR bombarding the top. Both the energy and depth dependence of the number of nucleons is given.

For a \( \gamma = 1 \) (normal) spectrum, \( \lambda N \) is just the average relative momentum \( \langle xL \rangle \) of nucleons produced in \( N \)-on-\( N \) collisions (see (112)). For \( \gamma > 1 \) (e.g. 1.7) the production of \( xL = 0 \) N's is suppressed in \( \lambda N \). The production of high-E N's is important as the push the Caca
Pions and Kaons in Atmospheric Cascades

We discuss π's. K's can be treated in identical fashion. The treatment of π's is more complicated as

i) they decay (this will be very important later as this decay is the primary source of CR muons and neutrinos)

\[ \pi^\pm \rightarrow \mu^\pm \]

ii) as nucleons they interact to produce more π's

iii) π's are also produced by nucleons

cascade eq. for pion flux \( \pi(x,E) \)

\[ \pi(E,x=0) = 0 \]

\[ \frac{d\pi(E,x)}{dx} = \left(-\frac{1}{\lambda_\pi} - \frac{1}{d\pi} \right) \pi \]

\[ + \frac{\pi}{\lambda_\pi} \int_0^1 dx_L x_L^{\theta-1} \left[ \frac{x_L}{\sigma_\pi} \frac{d\sigma_{\pi\pi}}{dx_L} \right] \pi \]

\[ + \frac{N}{\lambda_N} \int_0^1 dx_L x_L^{\theta-1} \left[ \frac{x_L}{\sigma_N} \frac{d\sigma_{N\pi}}{dx_L} \right] N \]

\[ \pi' = -\frac{\pi}{\lambda_\pi} - \frac{\pi}{d\pi} + \frac{\pi}{\lambda_\pi} \pi_{\pi\pi} + \frac{N}{\lambda_N} \pi_{N\pi} \]
\( \pi \)’s and \( K \)’s (cont)

E.g., let us take the high \( E \) limit. High energy \( p \)’s have an increased lifetime, so they travel distances \( \tau \)’s - large compared to the atmosphere (\( \tau \approx 10 \) km). We can neglect \( \pi/\pi \) term and (117) can be written

\[
\frac{d\pi}{dx} = -\frac{\pi}{\Lambda_\pi} + \frac{N}{\lambda_\pi^{'}} \zeta_{N\pi\pi} = -\frac{\pi}{\Lambda_\pi} + \frac{N_0}{\frac{\Lambda_{N\pi}}{\lambda_\pi}} \zeta_{N\pi\pi} + \frac{l}{N_0} \frac{\zeta_{N\pi\pi}}{\lambda_\pi}
\]

\[
\left\{ \begin{array}{l}
\Lambda_{\pi}^{-1} = \lambda_\pi^{-1} (1 - \zeta_{\pi\pi\pi}) \\
N = N_0(E) \exp \left(-\frac{\pi}{\Lambda_\pi}\right) \quad \text{(see 116)}
\end{array} \right.
\]

\( g(0) \) \( \exp(\varphi + t) \) and \( \Lambda^{-1}_N = \lambda^{-1}_N (1 - \zeta_{N\pi}) \)

**solution:**

\[ \pi = e^{-\frac{\pi}{\Lambda_\pi}} \alpha \int dx' \ e^{-\frac{\pi}{\Lambda_\pi}'(\frac{\lambda_\pi'}{\lambda_\pi} - \frac{\lambda_\pi}{\lambda_\pi'})} \]

Substitute two factors

\[ \frac{d\pi}{dx} = -\frac{1}{\pi} \pi + \exp \left(\frac{\pi}{\Lambda_\pi} \alpha \left(e^{\frac{\pi}{\Lambda_\pi'}} e^{-\frac{\pi}{\Lambda_\pi}}\right)\right) \]

\[ \pi = \alpha \ e^{-\frac{\pi}{\Lambda_\pi}} \left(\frac{1}{\Lambda_\pi} - \frac{1}{\lambda_\pi}\right)^{-1} \left[e^{\frac{\pi}{\Lambda_\pi}} e^{-\frac{\pi}{\Lambda_\pi-1}} \right] \]

\[ \pi = N_0 \frac{\zeta_{N\pi\pi}}{\lambda_\pi^{'}} \frac{\Lambda_{\Lambda_\pi} \Lambda_N}{\Lambda_N^{'}} \left[e^{\frac{\pi}{\Lambda_\pi}} e^{-\frac{\pi}{\Lambda_\pi-1}} \right] \]

\[ \pi = N N_0 \frac{\zeta_{N\pi\pi}}{\lambda_\pi^{'}} \frac{\Lambda_{\Lambda_\pi} \Lambda_N}{\Lambda_N^{'}} \left[e^{\frac{\pi}{\Lambda_\pi}} e^{-\frac{\pi}{\Lambda_\pi-1}} \right] \]
INPUT: from accelerator data
(all $\lambda$, $\Lambda$'s in grams cm$^{-2}$) (for air)

\[
\begin{align*}
Z_{pp} &= 0.27 \\
Z_{pn} &= 0.034 \\
Z_{NN} &\approx 0.30 = Z_{\pi \pi} \\
Z_{p\pi^+} &= 0.046 \\
Z_{p\pi^-} &= 0.035 \\
Z_{N\pi} &\approx 0.081 \\
Z_{p\pi^0} &= 0.041 \\
Z_{pk^+} &= 0.0092 \\
Z_{pk^-} &= 0.0030 \\
Z_p(K^0\bar{K}0) &= 2 \times Z_{pk^-} = 0.0060
\end{align*}
\]

\[
\begin{align*}
\lambda_N &= 86 \\
\lambda_{\pi} &= 1.3 \lambda_N \\
\Lambda_N &= 123 \\
\Lambda_{\pi} &= 162
\end{align*}
\]

\[
\Lambda_i = \frac{\lambda_i}{1 - Z_{ij}}
\]

\[
\frac{1.3 \times 86}{1 - 0.30} = \frac{1.3 \times 86}{0.7} = \frac{111.8}{0.7} = 159.7
\]
Muons in Air Showers

Origin: \( \pi \rightarrow \mu \nu \) decay

\[
d\pi/dx = -\left( \frac{1}{\Lambda_\pi} \right) \pi + \frac{N_0(E) z_{N\pi}}{\lambda_N} e^{-z_{\pi}}
\]

\[\Lambda_\pi = \frac{\lambda_\pi}{1 - z_{\pi}}\]

\[\pi\text{-decay}\]

\[t_\pi = \frac{\tau_\pi}{1 - \sqrt{3} \frac{z_{\pi}}{c}} = \gamma \tau_\pi\]

\[
d\pi\over d\pi = -\frac{dE}{N_{\pi}cT_\pi} = -\frac{dE}{\gamma cT_\pi} \quad \text{(see Fig. III for notation)}
\]

atmosphere

\[d\alpha = \rho \, dl\]

\[
\alpha = \alpha_0 e^{-\frac{hv}{ho}}
\]

\[h_\nu = l \cos \theta\]

\[\alpha_0 \approx 1030 \text{ g/cm}^2\]

\[h_0 = 6.4 \text{ km (at sea level)}\]

really a function of height
Muons (cont.)

decay length

$\frac{1}{4} \text{cm}^2 \rightarrow d_\pi = \rho \left( \frac{8e}{c} \tau_\pi \right) \quad dx = \rho \, dl$

for atmosphere

$$\rho = -\frac{dx}{dl} = -\frac{1}{\cos \theta} \frac{dx}{dl} = -\frac{x_0}{\cos \theta} \frac{d}{dl} e^{-\frac{\lambda x_0}{\cos \theta}}$$

$$\rho = \left( \frac{x_0}{\cos \theta} \right) e^{-\frac{\lambda x_0}{h_0}} = \frac{x_0 \cos \theta}{h_0}$$

$$d_\pi = \frac{x_0 \cos \theta}{h_0} \frac{E}{m_\pi c^2} \quad c \tau_\pi = \frac{E \cos \theta}{\varepsilon_\pi}$$

$\pi^- \rightarrow d_\pi = \frac{E \cos \theta}{\varepsilon_\pi}$

$favors$ $large$ $zenith$ $angles$ $(!longer$ $in$ $atmosphere)$

characteristic decay energy

lifetime $\sim$ energy $E$

$\varepsilon_\pi = \frac{h_0 (m_\pi c^2)}{c \tau_\pi}$

high energy $\pi$'s live longer, $d_\pi$ longer, less likely to decay (that's interact).

$\varepsilon_\pi$: energy of close competition between decay and interaction.
Particle

\begin{align*}
\mu^- & \quad 1 \\
\pi^+ & \quad 115 \\
\pi^0 & \quad 35 \times 10^2 \\
K^+ & \quad 850 \\
K^- & \quad 1.2 \times 10^5 \\
K^0 & \quad 205
\end{align*}

\textit{\pi-cascade : solution}

from 120

\[ \pi' = -\left( \frac{1}{\lambda_\pi} + \frac{\delta}{\Delta} \right) + \eta e^{-\frac{x^2}{2\lambda^2}} \quad \text{(neglects } \pi \rightarrow \pi) \]

\[ \delta(E) = \frac{\pi}{d_\pi} \]

\[ \delta = \frac{E_\pi}{E \cos \theta} \]

\[ \eta(E) = \frac{N_0(E)}{2n_\pi} \]

\text{Solution}

\[ \pi = \eta e^{-\frac{\chi^2}{2\lambda^2}} \int_0^{\infty} e^{-\frac{x^2}{2\lambda^2}} e^{-\frac{\chi^2}{2\lambda^2}} \left( \frac{\chi'}{\chi} \right) \delta \, dx' \]

\text{Proof: by calculating } d\pi/dx \text{ relative to factors circled.}
**Muons (cont)**

\[ \pi' = -\frac{\pi}{\lambda_\pi} + \eta e^{\frac{x}{\lambda_\pi}} e^{\frac{x'}{\lambda_\pi}} \left( \delta \left( \frac{x'}{x} \right) \right) \]

\[ + \eta e^{-\frac{x}{\lambda_\pi}} \int_{0}^{\infty} dx e^{\frac{x}{\lambda_\pi}} e^{-\frac{x'}{\lambda_\pi}} \left( \delta \left( \frac{x'}{x} \right) \delta \left( \frac{x'}{x-1} \right) \right) \delta \]

\[ - \frac{\sigma}{x} \pi \checkmark \]

**Other form:**

\[ \pi(E,x) = \int_{0}^{x} \frac{d\pi}{dx', x'} P_{0}(x-x') \, dx' \]

\[ P_{0}(x-x') = \left( \frac{x'}{x} \right) \delta e^{-\frac{(x-x')}{\lambda_\pi}} \]

\[ \rho : \text{probability that a } \pi \text{ produced at } x' \text{ survives to depth } x. \]

**High Energy limit:** \( \sigma \pi \to \infty, \frac{1}{d\pi} \to 0 \) mod decay

**Low Energy limit:** \( \delta \text{ very large, all } \pi \text{'s decay} \)

\[ \left( \frac{x'}{x} \right) \delta \to 0 \text{ as } \frac{x'}{x} < 1 \]

except for \( x' = x \)

\[ \pi(E,x) = \eta e^{-\frac{x}{\lambda_\pi}} e^{\frac{x'}{\lambda_\pi}} e^{-\frac{x'}{\lambda_\pi}} \left( \frac{x'}{x} \right) \delta \]

\[ \frac{\pi}{\delta+1} \left( \frac{\pi'}{\lambda_\pi} \right)^{\delta+1} \int_{0}^{\infty} \left( \frac{x'}{x} \right) \delta \]

\[ \left( \frac{x'}{x} \right) \delta+1 \]

\[ \rho \approx \frac{\alpha}{\delta+1} = \frac{\alpha}{\delta+1} \approx \frac{\alpha}{\delta} = \pi_{\text{av}} \]
solution when all \( \pi \)'s decay

\[
\pi(E,x) = \frac{d_{\pi} \eta}{\lambda_N} e^{-\frac{x}{\lambda_N}}
\]

\[
\pi(E,x) = \frac{d_{\pi} N_0 \pi_{\text{min}}}{\lambda_N} e^{-\frac{x}{\lambda_N}}
\]

\( \pi \) decay \( \rightarrow \) \( \mu \nu \)

\[
\frac{dP_{\mu}}{dx} = \int_{E_{\text{min}}}^{E_{\text{max}}} \frac{\pi(E_{\pi},x)}{d_{\pi}} \frac{dN_{\pi \rightarrow \mu}}{dE_{\mu}} (E_{\mu},E_{\pi}) dx
\]

from 2-body decay

\( \mu \) uniform angular distribution

projection

\[
\frac{dN}{dx} = \frac{2}{\pi} \frac{E_{\mu}}{x} = \frac{\alpha^2}{x}
\]

\( \alpha = E_{\mu} / E_{\pi} \)

energy \( \int_0^x \) all energy in \( \mu \)
**Muons (ctd)**

\[ B(\pi \to \mu \nu) = \int_0^1 \frac{dN}{dx} \, dx = cte \]

\[ \therefore \quad \frac{dN}{dx} = B(\pi \to \mu \nu) = 1 \]

\[ \frac{dN}{dE_\mu} = \frac{E_\pi}{E_\mu} B(\pi \to \mu \nu) = \frac{E_\pi}{E_\mu} \]

This assumes \( m_\mu = 0 \), correct answer

\[
\frac{dN_{\pi \to \mu}}{dE_\mu} = \frac{1}{\xi_\pi E_\pi}
\]

\[
\xi_\pi = 1 - \frac{m_\mu^2}{m_\pi^2}
\]

\[ \rightarrow \infty \text{ for } \xi \to 1 \]

\[
\frac{dP_\mu}{dx}(E_\mu, x) = \int_{\frac{1}{1 - \xi}}^{\frac{1}{\xi}} \frac{\pi(E_\pi, x)}{dE_\pi} \frac{1}{\xi \pi E_\pi} \, dE_\pi
\]
Use (125) in conjunction with $\pi(E,x)$ of (124). i.e. muons spectrum when all $\pi$'s decay

$$
\frac{dP_\mu}{dx} = \int \frac{1}{E_\mu} \frac{dE_\pi}{d\pi} \left( \frac{1}{\eta} e^{-\frac{2x}{\lambda N}} \right) \frac{1}{5\pi} \frac{1}{E_\pi} 
$$

$$
z = \frac{E_\pi}{E_\mu} \left( = \frac{1}{x} \right) 
$$

$$
\frac{dP_\mu}{dx} = \frac{1}{5} \int \frac{\pi(zE_\mu, x)}{d\pi} \frac{dz}{z} 
$$

$$
= \frac{\pi_{NP} e^{-\frac{2x}{\lambda N}}}{\frac{5}{\lambda N}} \int N_0(zE_\mu) \frac{dz}{z} 
$$

$$
= \frac{N_0(\frac{E_\mu}{z}) z^{-\frac{2x}{\lambda N}}}{\frac{5}{\lambda N}} \int \frac{dz}{z^{-(3+2)}} 
$$

int. over $x$

$$
\frac{1}{z^{3+1}} \left[ 1 - (1-\frac{2x}{\lambda N})^{x+1} \right] 
$$

$$
P_\mu(E_\mu, x) = \frac{N_0(E_\mu) \pi_{NP}}{\lambda N} \frac{\lambda N}{x^{3+1}} \left[ 1 - (1-\frac{2x}{\lambda N})^{x+1} \right] 
$$