Muon Monte Carlo: a new high-precision tool for muon propagation through matter

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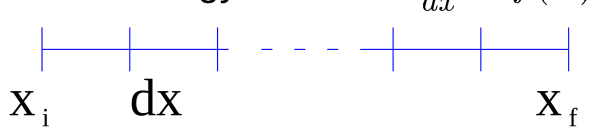
Derivation of tracking formulae

Continuous part of the energy losses: $-\frac{dE}{dx} = f(E)$



The stochastic part: $dP(E(x_i)) = \sigma(E(x_i))dx$. Probability to suffer a catastrophic loss on dx at x_f is

$$(1-dP(E(x_i)))\cdot\ldots\cdot(1-dP(E(x_f)))\cdot dP(E(x_f))$$

$$=\exp(-dP(E(x_i)))\cdot\ldots\cdot\exp(-dP(E(x_f)))\cdot dP(E(x_f))$$

$$=\exp\left(-\int_{E_i}^{E_f} dP(E(x))\right)\cdot dP(E(x_f))$$

$$=d_f\left(-\exp\left(-\int_{E_i}^{E_f}\frac{\sigma(E)}{-f(E)}\cdot dE\right)\right)=d(-\xi),\quad \xi\in(0;1]$$

The above equation can be solved for E_f:

$$\int_{E_i}^{E_f}\frac{\sigma(E)}{-f(E)}\cdot dE = -\log(\xi) \quad \text{(energy integral)},$$

and then the corresponding displacement is found:

$$x_f = x_i - \int_{E_i}^{E_f}\frac{dE}{f(E)} \quad \text{(tracking integral)}.$$

Continuous randomization

choose dx so small that $p_0=\int_{e_0}^{e_{cut}} p(e;E)\,de\cdot dx \ll 1$

$$<e>=\int_{e_0}^{e_{cut}} e\cdot p(e;E)\,de\cdot dx \qquad <e^2>=\int_{e_0}^{e_{cut}} e^2\cdot p(e;E)\,de\cdot dx$$

$$<(\Delta e)^2>=<e^2>-<e>^2$$

For small e_{cut}, $p(e;E)\simeq p(e;E_i)\rightarrow$ distributions $p(e;E)$ are the same $\rightarrow$ central limit theorem can be applied

$$<(\Delta(\Delta E))^2>=\sum_n\left(<e_n^2>-<e_n>^2\right)=$$

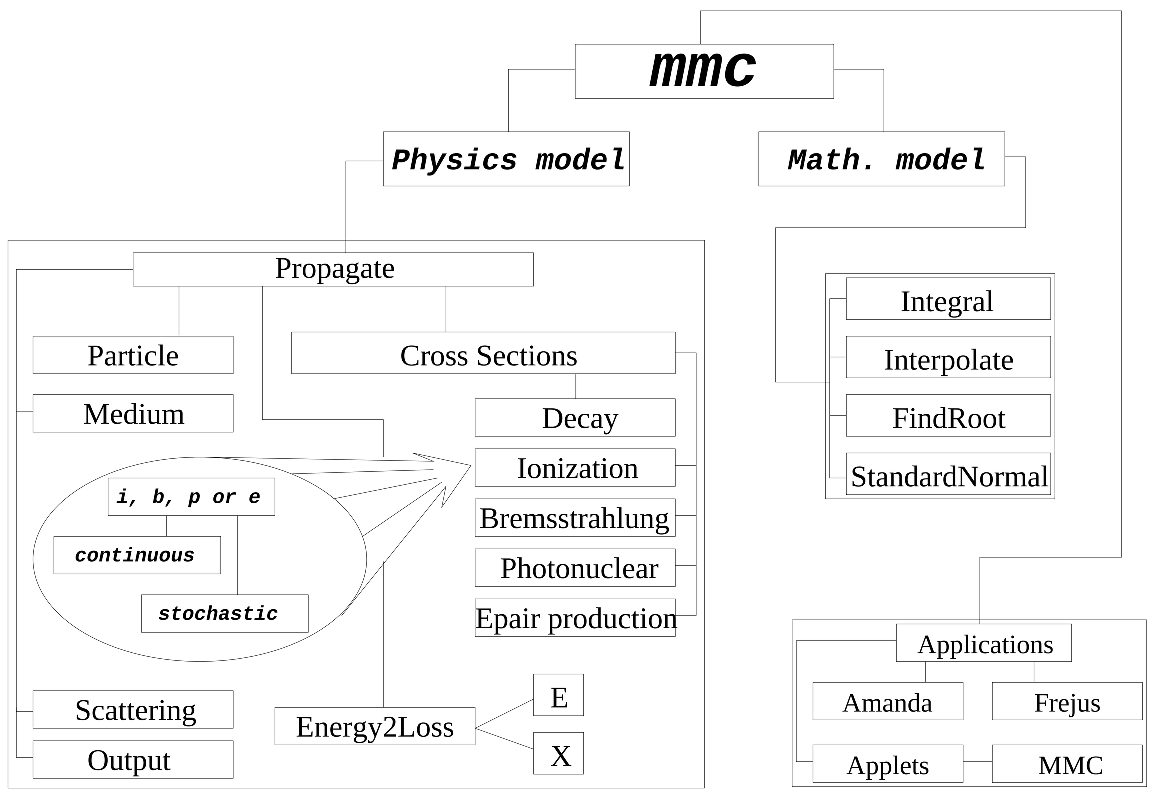
$$\sum_n\left[\left(\int_{e_0}^{e_{cut}} e_n^2\cdot p(e_n;E_i)\,de_n\right)dx_n-\left(\int_{e_0}^{e_{cut}} e_n\cdot p(e_n;E_i)\,de_n\right)^2 dx_n^2\right]$$

$$\simeq\int_{x_i}^{x_f} dx\cdot\left[\left(\int_{e_0}^{e_{cut}} e^2\cdot p(e;E(x))\,de\right)-\left(\int_{e_0}^{e_{cut}} e\cdot p(e;E(x))\,de\right)^2 dx\right]$$

Here E_i was replaced with average expectation value of energy at x, $E(x)$. As $dx\rightarrow 0$, the second term disappears. The lower limit of the integral over e can be replaced with zero, since all of the cross sections diverge slower than $1/e^3$. Then,

$$<(\Delta(\Delta E))^2>\simeq\int_{x_i}^{x_f}\frac{dE}{-f(E)}\cdot\left(\int_0^{e_{cut}} e^2\cdot p(e;E)\,de\right)$$

MMC structure

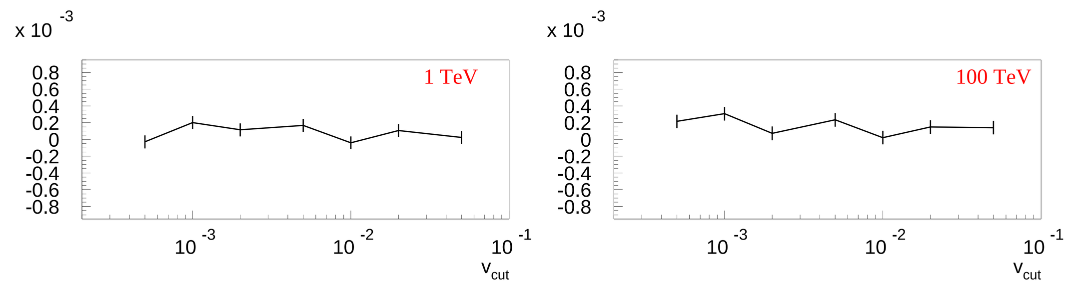


Results

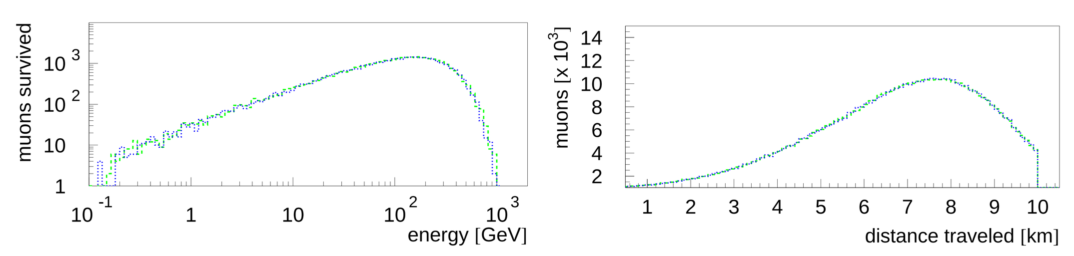
medium	a, $\frac{GeV}{mwe}$	b, $\frac{10^{-3}}{mwe}$	av. dev.	max. dev.
ice	0.259	0.357	3.7%	6.6%
fr. rock	0.231	0.429	3.0%	5.1%

medium	a, $\frac{GeV}{mwe}$	b, $\frac{10^{-3}}{mwe}$	av. dev.
ice	0.268	0.470	3.0%
fréjus rock	0.218	0.520	2.8%

Algorithm errors



Average final energy deviation from the purely "continuous" prediction

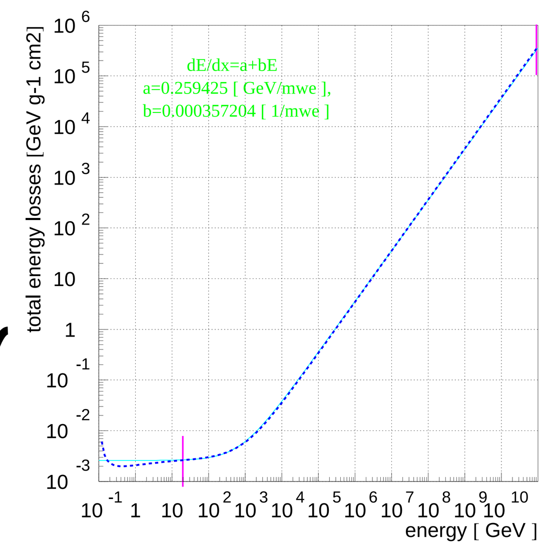


10^6 muons with energy 9 TeV propagated through 10 km of water: regular (dashed) vs. "cont" (dotted)

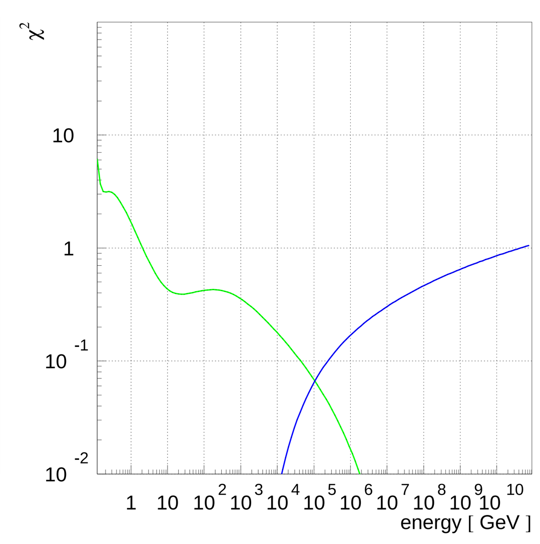
Survival probabilities

v_{cut}	"cont"	1 TeV 3 km	9 TeV 10 km	10^6 TeV 40 km
0.2	no	0	0	0.153
0.2	yes	0.010	0.057	0.177
0.05	no	0	0.035	0.143
0.05	yes	0.045	0.039	0.139
0.01	no	0.030	0.037	0.142
0.01	yes	0.034	0.037	0.139
10^{-3}	no	0.034	0.037	0.140
10^{-3}	yes	0.034	0.037	0.135

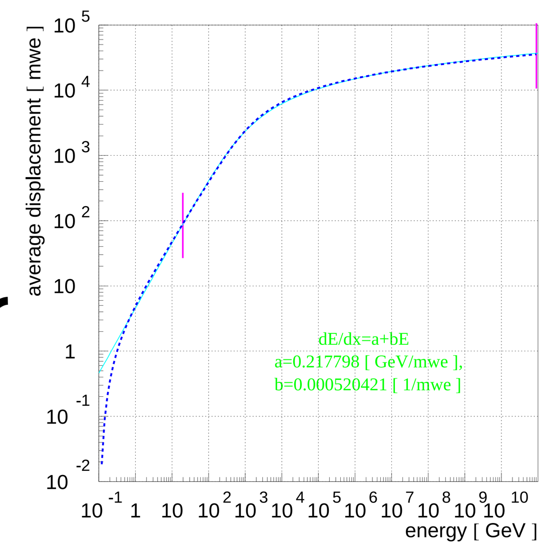
Fits to total energy loss and average range



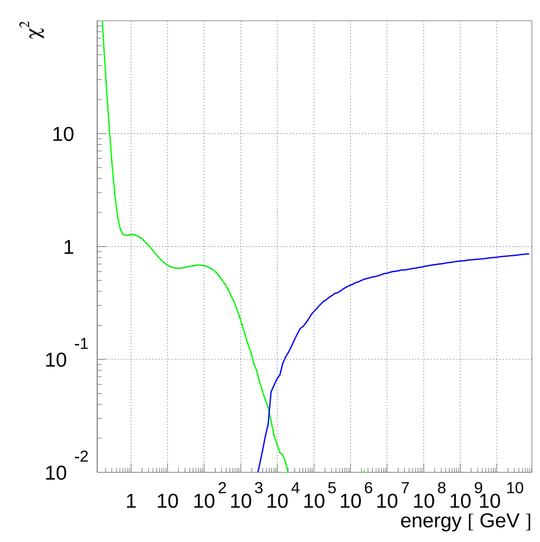
Fit to the energy losses in ice



χ^2 plot for energy losses in ice

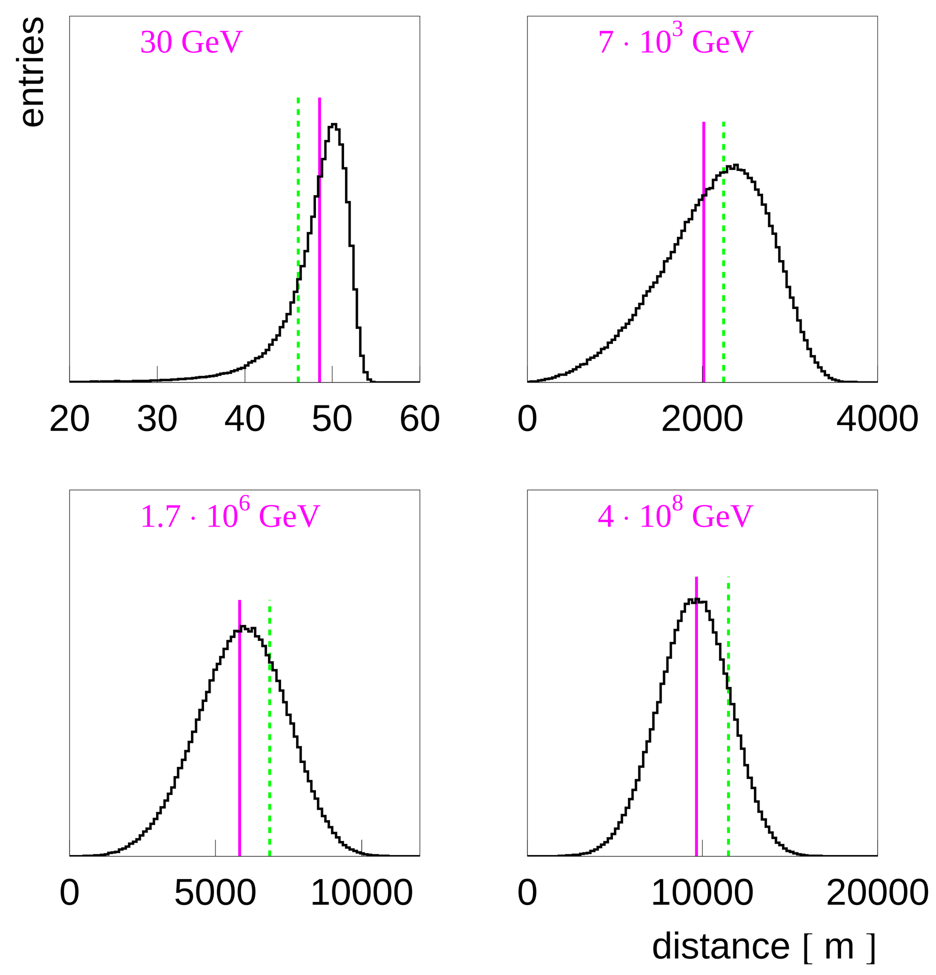


Fit to the average range in Fréjus rock



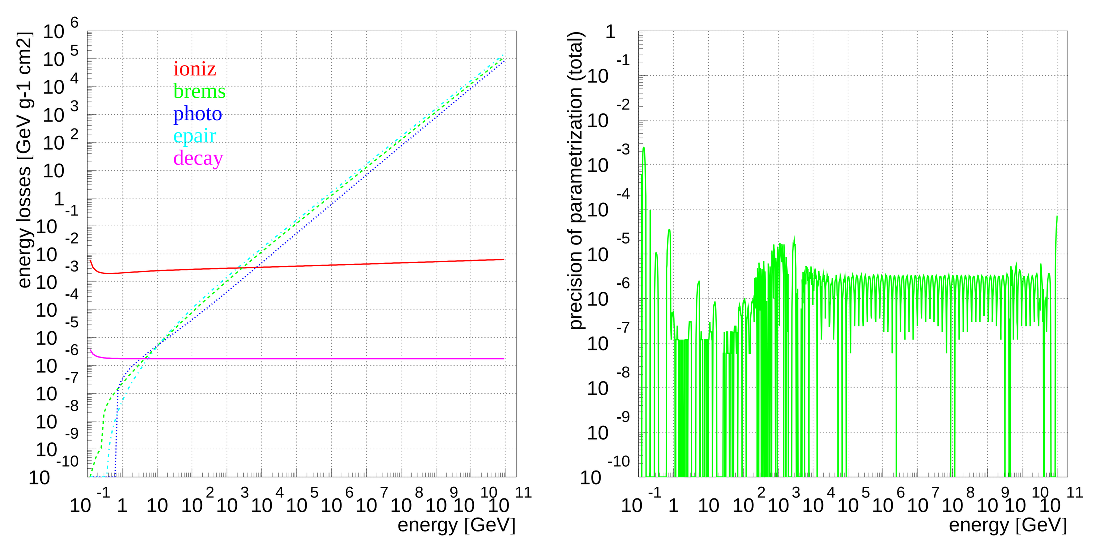
χ^2 plot for average range in Fréjus rock

Range distributions



in Fréjus rock: solid line designates the value of the range evaluated with the second table (continuous and stochastic losses) and the broken line shows the range evaluated with the first table (continuous losses only).

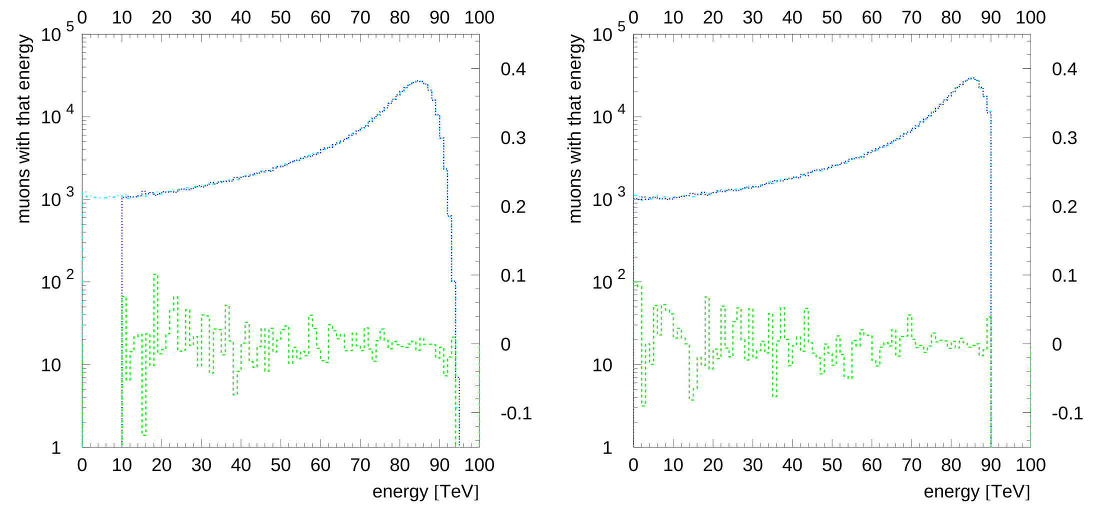
Parametrization errors:



energy losses in ice

Interpolation precision $(e_{pa}-e_{pa})/e_{pa}$

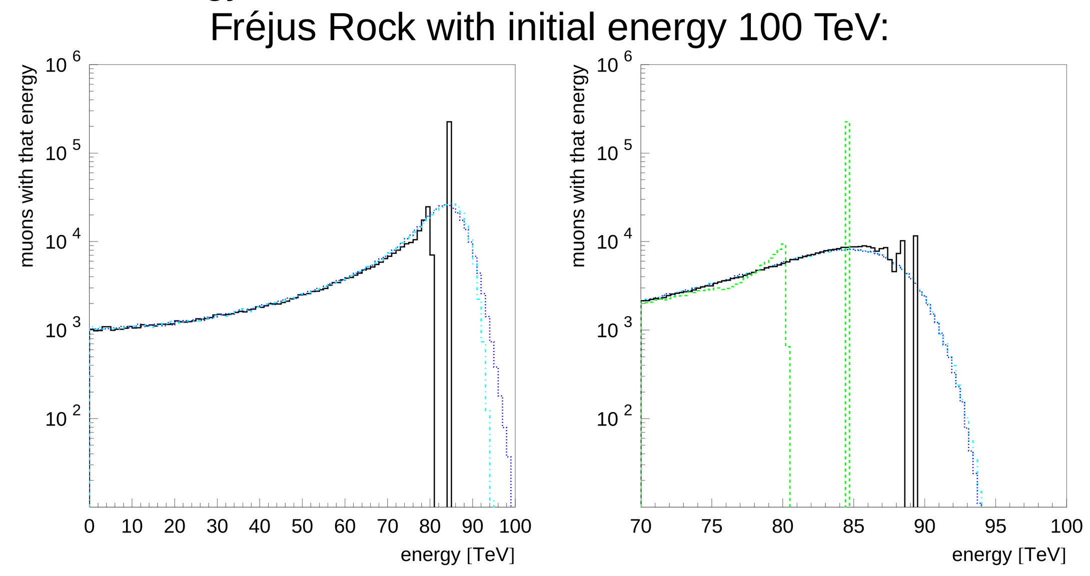
Final energy distribution of the muons that crossed 300 m of Fréjus Rock with initial energy 100 TeV:



Comparison of $e_{low}=m_\mu$ (dotted-dashed) with e_{low}=10 TeV (dotted). Also shown is the relative difference of the curves.

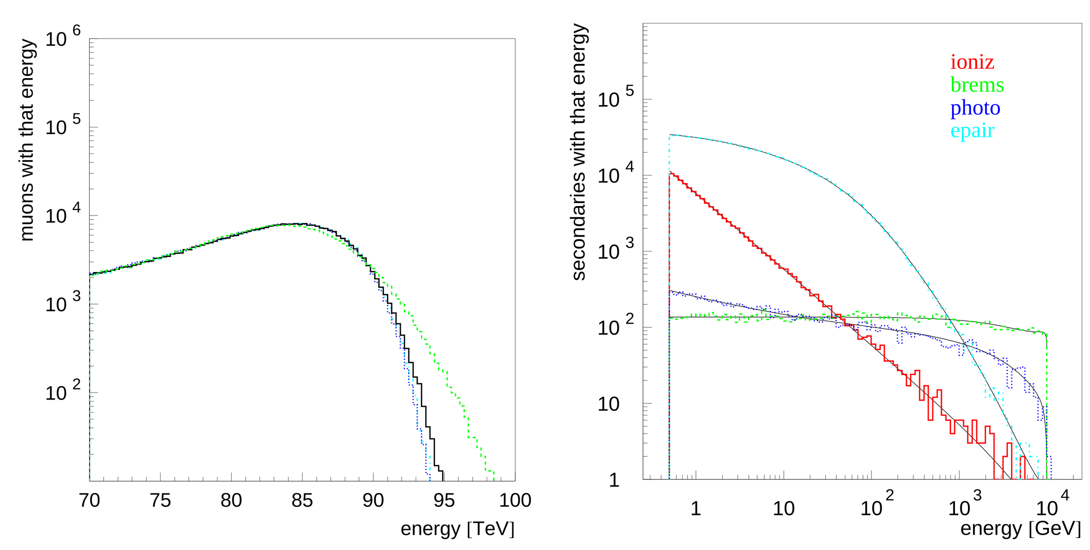
Comparison of (dashed-dotted) with exact (non-parametrized, dotted) versions for $v_{cut}=0.01$.

Final energy distribution of the muons that crossed 300 m of Fréjus Rock with initial energy 100 TeV:



$v_{cut}=0.05$ (solid), $v_{cut}=10^{-4}$ (dashed-dotted), $v_{cut}=0.05$ and "cont" option (dotted)

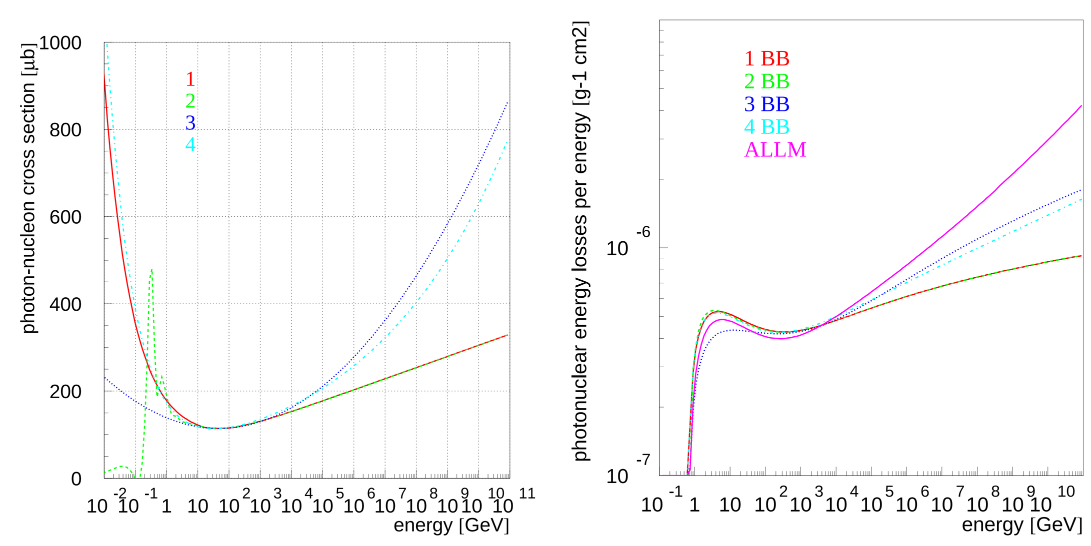
no "cont": $v_{cut}=0.05$ (dashed), $v_{cut}=0.01$ (solid), $v_{cut}=10^{-3}$ (dotted), $v_{cut}=10^{-4}$ (dashed-dotted)



with "cont" option: $v_{cut}=0.05$ (dashed), $v_{cut}=0.01$ (solid), $v_{cut}=10^{-3}$ (dotted), $v_{cut}=10^{-4}$ (dashed-dotted)

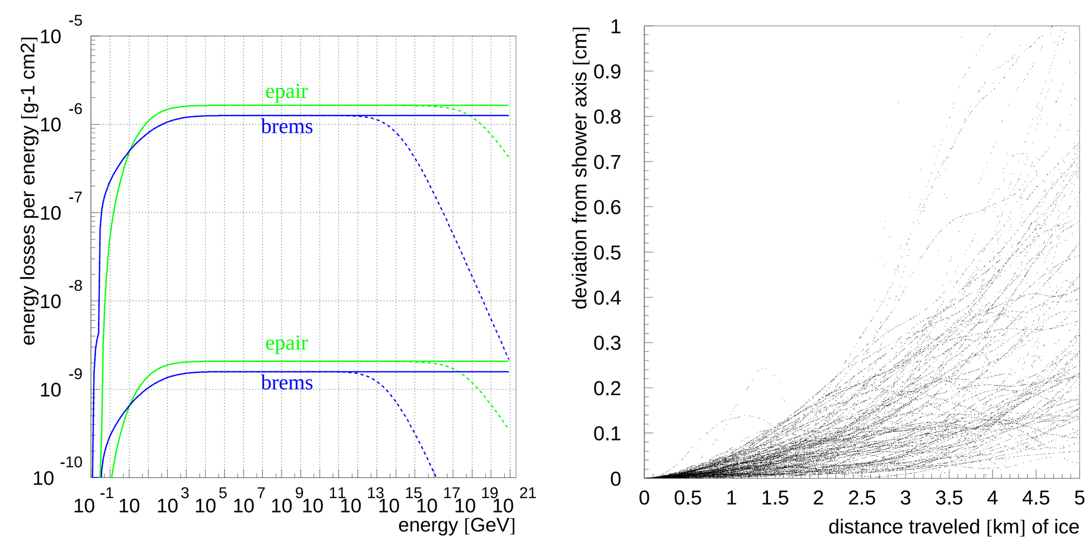
ioniz (upper solid curve), brems (dashed), photo (dotted), epair (dashed-dotted) spectra for E_μ=10 TeV in the Fréjus rock

Photonuclear losses, LPM effect and Molière scattering



different photon-nucleon cross sections implemented in MMC, Bezrukov Bugaev parametrization

photonuclear energy losses (divided by energy), according to different formulae.



LPM effect in ice (higher plots) and Fréjus rock (lower plots, multiplied by 10^{-3})

Molière scattering of 100 10 TeV muons going straight down through ice