Name KEY

Exam #2

Physics 248

March 14, 2007

Each problem is worth 25 points

Problem	Score
1	25
2	25
3	25
4	25
Total	100

Physical constants: $h = 6.626 \times 10^{-34} Js$ | $c = 2.998 \times 10^{8} m/s$ | 1 eV$=1.602 \times 10^{-19} J$ | $\hbar c = 1973 eV\AA$

1. **Multiple choice question** (circle your answer):

Consider two sources S_1 and S_2 emitting waves with equal amplitude A, frequency $f = 2$ Hz and velocity $v = 3\text{m/s}$. Sources S_1 and S_2 have the same phase.

(i) What is the wavelength λ?

(A) 6 m (B) 2/3 m (C) 3/2 m (D) 1/6 m (E) 3π m

(ii) Suppose the wave source S_1 is 5 m directly to your right while S_2 is 5.75 m directly to your left, the result is

(A) a wave with the same frequency but twice the amplitude
(B) a wave with the same amplitude but twice the frequency
(C) a wave with twice the frequency and twice the amplitude
(D) a wave with zero amplitude
(E) a wave with an intensity equal to the sum of the intensities of the two waves

(iii) Now, increase the amplitude of S_1 to $2A$ while keeping the amplitude of S_2 fixed, the amplitude of the resulting wave is

(A) zero (B) A (C) 2 A (D) 3A (E) 9A

2. A new particle, Montarulino, was recently discovered at an accelerator in Bari, Italy. The rest mass of Montarulino is 1 TeV/c^2. Usually, particles at accelerators are highly energetic, but Jim the God on Earth found a way to slow Montarulino down to a kinetic energy of 10 eV.

(i) What is the wavelength of the slow Montarulino in $\mathring{A}$?

$10\,eV \ll 1\,TeV \Rightarrow$ Nonrelativistic

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{2\pi\hbar c}{\sqrt{2mc^2E}}$$

$$= \frac{2\pi(1973\,eV\,\mathring{A})}{\sqrt{2\cdot 10^{12}eV\cdot 10eV}}$$

$$= \boxed{2.77\times 10^{-3}\,A}$$

(ii) Jim has a misguided theory that nuclei ($\sim 10^{-15}m$) consist of a regular array of nucleons (protons and neutrons). Would Jim be able to test this theory by measuring the diffraction pattern of his slow Montarulinos incident on nuclei? Explain your answer.

$d\sin\theta = n\lambda$; $d \sim 10^{-15}m$

$\lambda \gg d \Rightarrow \frac{\lambda}{d} \gg 1$ for 1st diffraction maximum $\Rightarrow$ $\boxed{\text{diffraction not possible}}$

3. In a different universe, the electric potential between an electron and a proton in a "hydrogen atom" is

$$V(r) = -ke^2/r^{n+1}$$

where $n > 0$, instead of the familiar Coulomb potential $V(r) = -ke^2/r$ we discussed in class. Use uncertainty principle arguments to estimate the ground state energy. Don't worry about the overall numerical factor.

$$\Delta p \Delta x \geq \hbar$$

$$p \sim \Delta p \geq \frac{\hbar}{\Delta x} \sim \frac{\hbar}{r}$$

$$E = KE + V$$

$$= \frac{p^2}{2m} - \frac{ke^2}{r^{n+1}}$$

$$\simeq \frac{\hbar^2}{2mr^2} - \frac{ke^2}{r^{n+1}}$$

Minimize $E \Rightarrow \frac{dE}{dr} = 0$

$$0 = \frac{dE}{dr} = -\frac{\hbar^2}{mr^3} + \frac{(n+1)ke^2}{r^{n+2}}$$

$$\frac{\hbar^2}{mr^3} = \frac{(n+1)ke^2}{r^{n+2}} \Rightarrow r^{(n-1)} = \frac{(n+1)ke^2 m}{\hbar^2}$$

$$r = \left[\frac{(n+1)ke^2 m}{\hbar^2}\right]^{\frac{1}{(n-1)}}$$

Solve for E:

$$E = \frac{\hbar^2}{2mr^2} - \frac{ke^2}{r^{n+1}}$$

$$\boxed{= \frac{\hbar^2}{2m\left[\frac{(n+1)ke^2 m}{\hbar^2}\right]^{2/(n-1)}} - ke^2\left[\frac{(n+1)ke^2 m}{\hbar^2}\right]^{-\frac{(n+1)}{(n-1)}}}$$

4. The wavefunction for a particle of mass m is $\psi(x) = \psi_0 e^{-\frac{|x|}{2a}}$ for some constant a, and where $-\infty < x < +\infty$. You may find the following integral helpful: $\int_0^\infty x^n e^{-x} dx = n!$ for any non-negative integer n.

(i) Normalize this wavefunction. That is, find the coefficient ψ_0 such that the probability of finding the particle between $\pm\infty$ is one.

$$1 = \int_{-\infty}^{\infty} \psi_0^2 \left[e^{-|x|/2a}\right]^2 dx$$

$$= 2\psi_0^2 \int_0^\infty e^{-x/a} dx$$

$$= 2\psi_0^2 \cdot a \Rightarrow \boxed{\psi_0 = \frac{1}{\sqrt{2a}}}$$

(ii) Assuming the total energy of the particle is $E = -\frac{\hbar^2}{8ma^2}$, use the time independent Schrodinger equation to show that $V(x)$ is constant for $x > 0$ and $x < 0$, and find this constant. Don't worry about $V(x)$ at $x = 0$.

$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + V\psi = E\psi$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\left(\frac{1}{\sqrt{2a}}e^{-|x|/2a}\right)$$

$$= \frac{-\hbar^2}{2m}\cdot\frac{1}{4a^2}\cdot\frac{1}{\sqrt{2a}}e^{-|x|/2a}$$

$$= \frac{-\hbar^2}{8ma^2}\cdot\psi = E\psi \quad \text{Since } E = \frac{-\hbar^2}{8ma^2} \Rightarrow \boxed{V(x) = 0}$$

(iii) Calculate $< x^2 >$ and $< x >^2$.

$$\langle x\rangle^2 = \left[\int_{-\infty}^{\infty} x \cdot \frac{1}{2a}\left(e^{-|x|/2a}\right)^2 dx\right]^2 = \boxed{0} \text{ by symmetry}$$

$$\langle x^2\rangle = \int_{-\infty}^{\infty} x^2 \cdot \frac{1}{2a}\left(e^{-|x|/2a}\right)^2 dx$$

$$= \frac{1}{a}\int_0^\infty x^2 e^{-x/a} dx$$

$$= \frac{1}{a}\int_0^\infty a^2 u^2 e^{-u}\, a\, du$$

$$= a^2 \int_0^\infty u^2 e^{-u} du$$

$$= \boxed{2a^2}$$