Exam #2  
Physics 248  
March 14, 2007  

Each problem is worth 25 points

<table>
<thead>
<tr>
<th>Problem</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
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<tr>
<td>4</td>
<td>25</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
</tr>
</tbody>
</table>

Physical constants: \( h = 6.626 \times 10^{-34} \text{Js} \) \( c = 2.998 \times 10^8 \text{m/s} \) \( 1 \text{eV} = 1.602 \times 10^{-19} \text{J} \) \( hc = 1973 \text{eV}\text{Å} \)
1. **Multiple choice question** (circle your answer):

Consider two sources $S_1$ and $S_2$ emitting waves with equal amplitude $A$, frequency $f = 2$ Hz and velocity $v = 3$ m/s. Sources $S_1$ and $S_2$ have the same phase.

(i) What is the wavelength $\lambda$?

(A) 6 m  (B) 2/3 m  (C) 3/2 m  (D) 1/6 m  (E) 3\pi m

(ii) Suppose the wave source $S_1$ is 5 m directly to your right while $S_2$ is 5.75 m directly to your left, the result is

(A) a wave with the same frequency but twice the amplitude  
(B) a wave with the same amplitude but twice the frequency  
(C) a wave with twice the frequency and twice the amplitude  
(D) a wave with zero amplitude  
(E) a wave with an intensity equal to the sum of the intensities of the two waves

(iii) Now, increase the amplitude of $S_1$ to $2A$ while keeping the amplitude of $S_2$ fixed, the amplitude of the resulting wave is

(A) zero  (B) $A$  (C) 2A  (D) 3A  (E) 9A
2. A new particle, Montarulino, was recently discovered at an accelerator in Bari, Italy. The rest mass of Montarulino is $1 \text{TeV}/c^2$. Usually, particles at accelerators are highly energetic, but Jim the God on Earth found a way to slow Montarulino down to a kinetic energy of $10 \text{eV}$.

(i) What is the wavelength of the slow Montarulino in Å?

\[
\frac{10 \text{eV}}{1 \text{TeV}} \Rightarrow \text{Nonrelativistic}
\]

\[
\lambda = \frac{\hbar}{p} = \frac{\hbar}{\sqrt{2mE}} = \frac{2\pi \hbar c}{\sqrt{2mE}} = \frac{2\pi (1973 \text{eV} \text{Å})}{\sqrt{2 \cdot 10^3 \text{eV} \cdot 10 \text{eV}}} = 2.77 \times 10^{-3} \text{Å}
\]

(ii) Jim has a misguided theory that nuclei (~ $10^{-15} \text{m}$) consist of a regular array of nucleons (protons and neutrons). Would Jim be able to test this theory by measuring the diffraction pattern of his slow Montarulinos incident on nuclei? Explain your answer.

\[
d \sin \theta = n \lambda \quad ; \quad d \sim 10^{-15} \text{m}
\]

\[
\lambda \gg d \Rightarrow \frac{\lambda}{d} \gg 1 \quad \text{for 1st diffraction maximum} \Rightarrow \text{diffraction not possible}
\]
3. In a different universe, the electric potential between an electron and a proton in a "hydrogen atom" is

\[ V(r) = -\frac{ke^2}{r^{n+1}} \]

where \( n > 0 \), instead of the familiar Coulomb potential \( V(r) = -\frac{ke^2}{r} \) we discussed in class. Use uncertainty principle arguments to estimate the ground state energy. Don't worry about the overall numerical factor.

\[
E = KE + V = \frac{p^2}{2m} - \frac{ke^2}{r^{n+1}} = \frac{\hbar^2}{2mr^2} - \frac{ke^2}{r^{n+1}}
\]

Minimize \( E \Rightarrow \frac{dE}{dr} = 0 \)

\[
0 = \frac{dE}{dr} = -\frac{\hbar^2}{mr^3} + \frac{(n+1)ke^2}{r^{n+2}}
\]

\[
\frac{\hbar^2}{mr^3} = \frac{(n+1)ke^2}{r^{n+2}} \Rightarrow r^{(n-1)} = \frac{(n+1)ke^2m}{\hbar^2}
\]

\[
r = \left[ \frac{(n+1)ke^2m}{\hbar^2} \right]^{\frac{1}{n-1}}
\]

Solve for \( E \):

\[
E = \frac{\hbar^2}{2mr^2} - \frac{ke^2}{r^{n+1}} = \frac{\hbar^2}{2m\left[ \frac{(n+1)ke^2m}{\hbar^2} \right]^{\frac{1}{(n-1)}}} - \frac{ke^2}{\left[ \frac{(n+1)ke^2m}{\hbar^2} \right]^{\frac{1}{(n-1)}} - \frac{(n+1)}{(n-1)}}
\]
4. The wavefunction for a particle of mass \( m \) is \( \psi(x) = \psi_0 e^{-|x|/2a} \) for some constant \( a \), and where \( -\infty < x < +\infty \). You may find the following integral helpful: \( \int_0^\infty x^n e^{-x} dx = n! \) for any non-negative integer \( n \).

(i) Normalize this wavefunction. That is, find the coefficient \( \psi_0 \) such that the probability of finding the particle between \( \pm \infty \) is one.

\[
1 = \int_{-\infty}^{\infty} \psi_0^2 \left( e^{-|x|/2a} \right)^2 dx \\
= 2\psi_0^2 \int_0^{\infty} e^{-x/a} dx \\
= 2\psi_0^2 \cdot a \\
\Rightarrow \psi_0 = \frac{1}{\sqrt{2a}}.
\]

(ii) Assuming the total energy of the particle is \( E = -\frac{h^2}{8ma^2} \), use the time independent Schrodinger equation to show that \( V(x) \) is constant for \( x > 0 \) and \( x < 0 \), and find this constant. Don’t worry about \( V(x) \) at \( x = 0 \).

\[
-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi + V \psi = E \psi \\
-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \left( \frac{1}{\sqrt{2a}} e^{-|x|/2a} \right) \\
= -\frac{\hbar^2}{2m} \cdot \frac{1}{4a^2} \cdot \frac{1}{\sqrt{2a}} e^{-|x|/2a} \\
= \frac{\hbar^2}{8ma^2} \psi = E \psi \quad \text{so} \quad E = -\frac{\hbar^2}{8ma^2} \Rightarrow V(x) = 0
\]

(iii) Calculate \( < x^2 > \) and \( < x^2 > \).

\[
<x^2> = \left[ \int_{-\infty}^{\infty} x^2 \left( \frac{1}{\sqrt{2a}} e^{-|x|/2a} \right)^2 dx \right]^2 = 0 \quad \text{by symmetry}
\]

\[
<x^2> = \int_{-\infty}^{\infty} x^2 \left( \frac{1}{\sqrt{2a}} e^{-|x|/2a} \right)^2 dx \\
= \frac{1}{a} \int_0^{\infty} x^2 e^{-x/a} dx \\
= \frac{1}{a} \int_0^{\infty} a^2 u^2 e^{-u} du \\
= a^2 \int_0^{\infty} u^2 e^{-u} du \\
= 2a^2
\]