Name Solutions

Exam #1

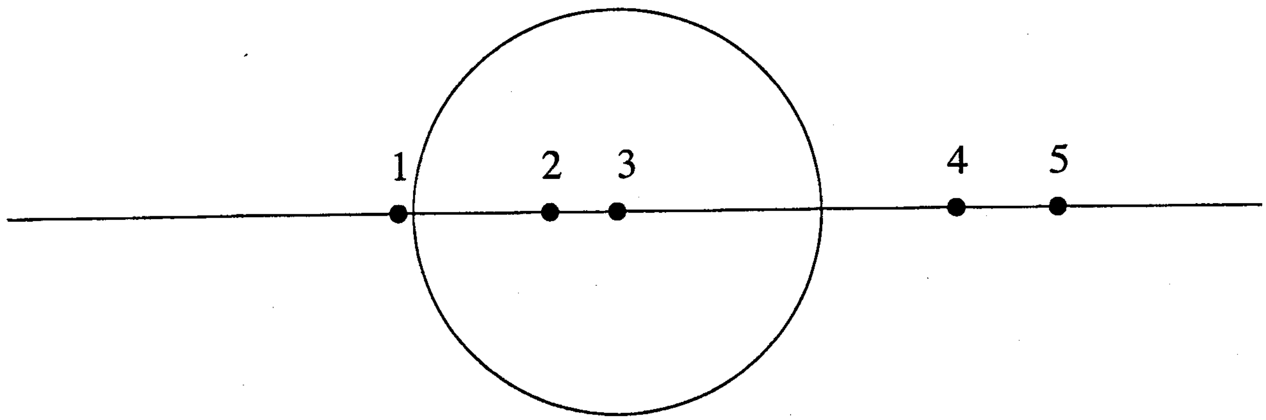
Physics 248

February 14, 2007

Each problem is worth 25 points

Problem	Score
1	
2	
3	
4	
Total	

1. **Multiple choice question** (circle your answer):



(i) If the circle represents a solid sphere with uniform mass density, and the line passes through the center, the gravitational field will have the greatest value at:

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

(ii) For the setup in (i), the gravitational field will have its least value at:

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

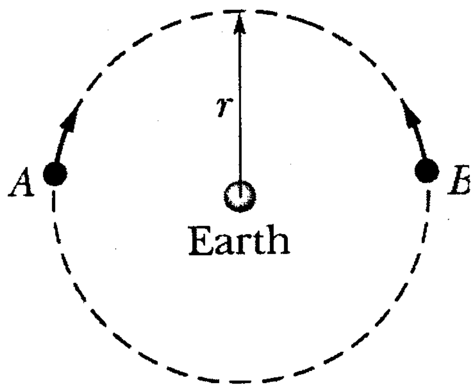
(iii) Replace the solid sphere with a thin spherical shell with uniform surface mass density. The gravitational field will have the greatest value at:

(A) 1 (B) 4 (C) 5 (D) 2,3, and 5 (E) 2 and 3

(iv) For the setup in (iii), the gravitational field will have the smallest value at:

(A) 1 (B) 4 (C) 5 (D) 2,3, and 5 (E) 2 and 3

2. Consider two satellites , A and B, both of mass m, moving in the same circular orbit of radius r around the earth, of mass M_E, but in opposite sense of rotation and therefore on a collision course (see figure).



(i) In terms of G, M_E, m, and r, find the total mechanical energy $E_A + E_B$ of the two-satellite-plus-Earth system.

$$E_A = -\frac{GM_E m}{r} + \frac{1}{2}mv^2 = -\frac{GM_E m}{2r}$$

$$E_A + E_B = 2 \times \left(-\frac{GM_E m}{2r}\right) = -\frac{GM_E m}{r}$$

(ii) If the collision is completely inelastic so that the wreckage remains as one piece of tangled material (mass=$2m$), find the total mechanical energy immediately after collision.

Since $v = 0 \Rightarrow K.E. = 0$

$$E = -\frac{GM_E (2m)}{r} = -\frac{2GM_E m}{r}$$

(iii) Describe the subsequent motion of the wreckage.

Falls towards the center of earth

3. For each term on the list on the left, indicate the term, description, or equation on the right that is most closely associated with it. Example, the Master of Physics would be Einstein so you would enter "a" in the blank beside that term. Note that a grade enhancement could result from entering "b" instead. You may use a term from the right only once on the left side.

n p Gravitational frequency shift formula
h Black hole
b a Master of Physics
g Deflection of light
l Principle of Equivalence

i Inertial mass
c Gravitational blue shift
p n Dimensionless quantity for gravity field around star

(a) Albert Einstein
(b) Jim Braun
(c) Light received on earth from satellite
(d) Time travel
(e) Light has no rest mass and therefore it does not fall in a gravitational field
(f) GMc^2/R^2
(g) Gravitational Lensing
(h) Schwarzschild radius
(i) Gravitational mass
(j) $E = mc^2$
(k) $\omega c^2/\Delta\phi$
(l) A uniform gravitational field is indistinguishable from a uniform acceleration
(m) Light received on earth from massive star
(n) $GM/(Rc^2)$
(o) Quadrupole mass oscillations
(p) $\omega\Delta\phi/c^2$

4. The inhabitants of the moon are jealous of us and want to develop their own Global Positioning System (GPS). Their GPS satellites are parked at such an elevation so as to revolve around the moon every 12 hours. Given the mass of the moon is $7.3 \times 10^{22} kg$: radius = 1737.4 km

(i) Calculate the satellite's speed v_s and radial distance r_s from the center of the moon.

$$\frac{V_s^2}{r_s} = \frac{GM}{r_s^2} \Rightarrow V_s^2 = \frac{GM}{r_s} \Rightarrow \left(\frac{2\pi r_s}{T}\right)^2 = \frac{GM}{r_s}$$

$$\therefore\ r_s = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} = 6.13\times10^6\ m$$

$$V_s = \frac{2\pi r_s}{T} = \cancel{1782.7\ m/s}\quad 891.1\ m/s$$

(ii) Calculate the fractional change in the time measured due to special relativistic time dilation.

$$\frac{\Delta t_{satellite}}{\Delta t_{earth}} = \left(1-\beta_s^2\right)^{-1/2} \simeq 1 + \frac{\beta_s^2}{2} + \cdots$$

$$\text{Fractional change} = \frac{\beta_s^2}{2} = \frac{1}{2}\left(\frac{V_s}{c}\right)^2 = \frac{1}{2}\left(\frac{1782.7}{3\times10^8}\right)^2 =$$

$$= \cancel{1.77\times10^{-11}}\quad 4.41\times10^{-12}$$

(iii) Calculate the fractional change in the time measured due to the gravitational time dilation effect.

$$\frac{\phi_{moon}-\phi_s}{c^2} = -\frac{GM}{c^2}\left(\frac{1}{R_{moon}}-\frac{1}{r_s}\right)$$

$$= -\frac{6.67\times10^{-11}\cdot 7.3\times10^{22}}{(3\times10^8)^2}\left[\frac{1}{1.7\times10^6}-\frac{1}{6.13\times10^6}\right]$$

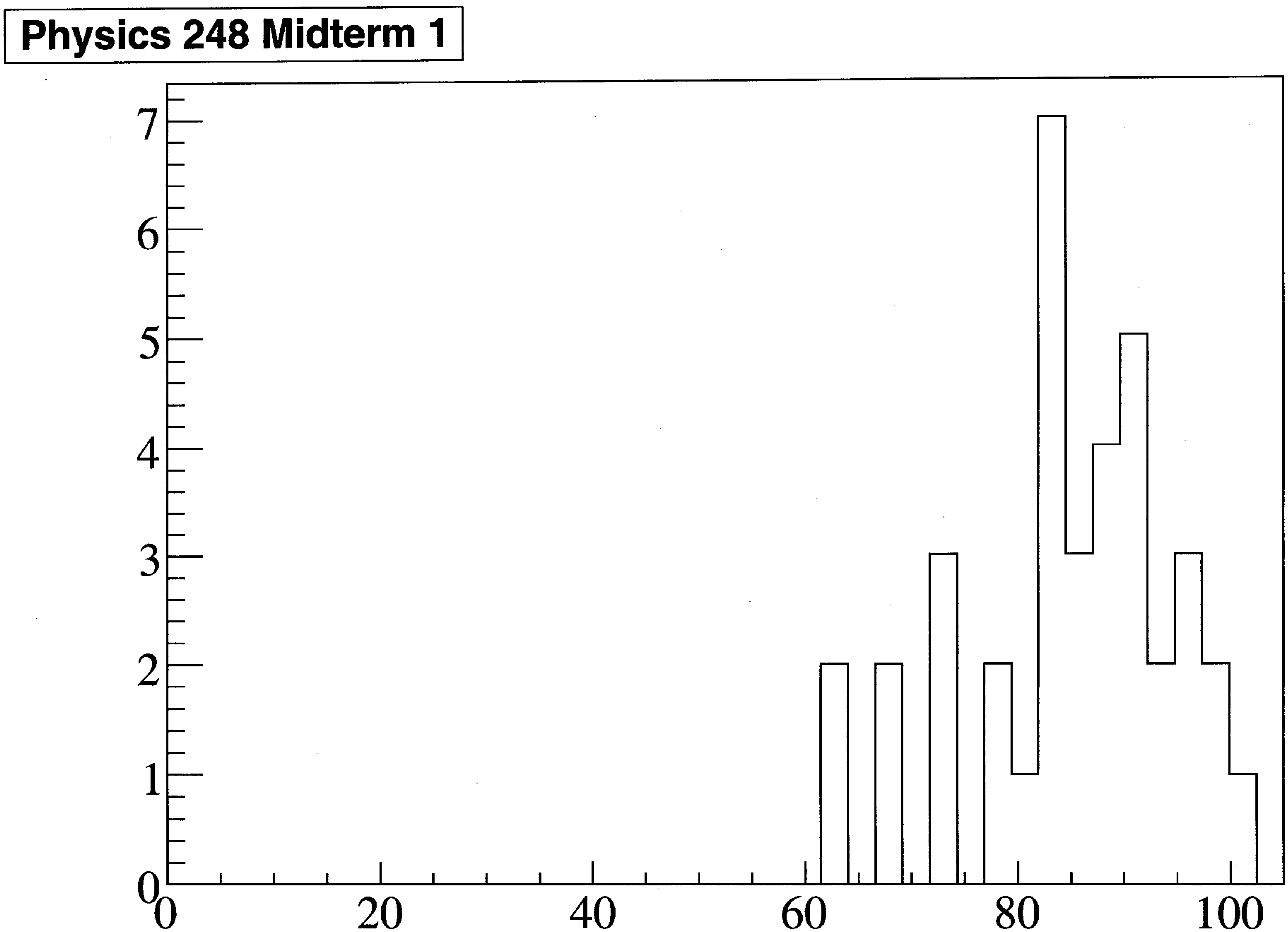
$$= -2.3\times10^{-11}\quad -2.23\times10^{-11}$$

(iv) Calculate the error that can be accumulated in 1 minute because of these relativistic corrections.

$$(\Delta t)_{GR} = -2.3\times10^{-11}\times 60s = \cancel{-1.38\times10^{-9}}\ s\quad -1.34\times10^{-9}\ s$$

$$(\Delta t)_{SR} = 1.77\times10^{-11}\times 60s = \cancel{1.06\times10^{-9}}\ s\quad 2.64\times10^{-10}\ s$$

$$(\Delta t)_{total} = \cancel{-0.32\times10^{-9}}\ s\quad -1.076\ ns$$



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