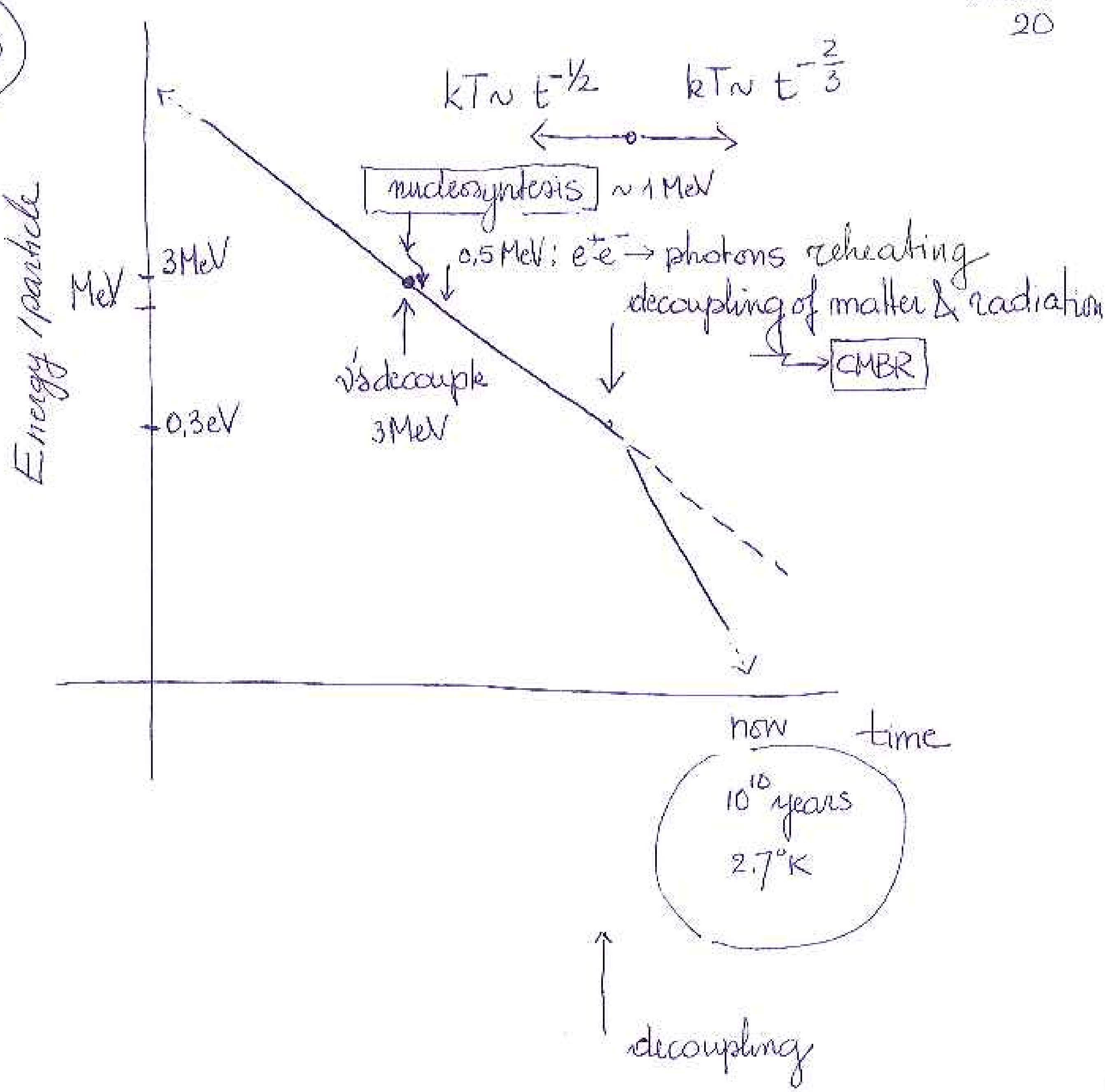
Overview



$$\rho_{m0}\left(\frac{R_0}{R_d}\right)^3 \simeq \rho_{r0}\left(\frac{R_0}{R_d}\right)^4$$

$$\frac{R_0}{R_d} \simeq \frac{\rho_{m0}}{\rho_{r0}} \simeq \frac{\Omega_b}{\Omega_{rad}} \simeq \frac{0{,}01}{5\times10^{-5}}$$

$$\simeq 2000$$

$$T_K = \frac{1.52\times10^{10}}{g^{1/4}\,t^{1/2}} = 5000\,^\circ K \quad \longleftarrow \quad t_d = \frac{12}{g^{1/2}\,E_{MeV}^2} \simeq 10^{13}\,sec$$

$g^{1/2}$: $\sqrt{4}$ $\qquad$ E_{MeV}: 0,3~13.6 eV

$$T_{K0} = \frac{R_d}{R_0}\,(5000\,^\circ K) = 2.5\,^\circ K \quad \checkmark$$

- nucleosynthesis

Universe at 1 MeV : $\underbrace{n, p}$ and $\overbrace{e^+, e^-, \gamma}^{\text{determine } H}$

$kT \ll m_N c^2 \longleftarrow E \simeq mc^2 + \frac{p^2}{2m}$

Fermi-Dirac distribution

$$pc^2 = \frac{h}{2\pi^2 \hbar^3} \int_0^\infty \frac{E\, p^2 dp}{e^{E/kT} + 1}$$

$$= \frac{h}{2\pi^2 \hbar^3} mc^2 e^{-mc^2/kT} \int dp\, p^2 e^{-\frac{p^2}{2mkT}}$$

$\int x^2 e^{-x^2} dx = \frac{1}{4}\sqrt{\pi}$

$$pc^2 = mc^2 \frac{h}{\hbar^3} \left(\frac{mkT}{2\pi}\right)^{3/2} e^{-mc^2/kT}$$

$$\boxed{r = \frac{n_n}{n_p} = e^{-\overbrace{(m_n - m_p)c^2}^{1.3\,\text{MeV}}/kT}}$$

weak

$n + \nu_e \leftrightarrow p + e^-$

$n + e^+ \longleftrightarrow p + \bar{\nu}_e$

stops at $T < m_e c^2 \simeq \frac{1}{2}$ MeV

em 0.1 mb

$p + n \rightarrow d + \gamma$ $+ 2.22$ MeV

$d + n \rightarrow {}^3H + \gamma$

$d + p \rightarrow {}^3He + \gamma$

${}^3He + p \rightarrow He^4 + \gamma$

${}^3He + n \rightarrow He^4 + \gamma$

once γ's cannot photodisintegrate d (binding energy 2.2 MeV), all n become ^4He

Universe is H, He^4.

- What is the final $\frac{n}{p}$ ratio?

1. do a freeze-out calculation for $n \leftrightarrow p$ weak transitions as we did for neutrino decoupling (these cross sections are smaller than $e^+e^- \leftrightarrow \nu+\bar{\nu}$ therefore nucleosyntesis starts a lower temperature). The answer is $kT = 0.80$ MeV. Therefore

$$\boxed{\frac{n}{p} = e^{-\frac{(m_n-m_p)c^2}{kT}} = 0.2}$$

2. neutrons decay $n \to n e^{-t/\tau}$ $\tau = 887$ s

3. the neutrons that have not decayed are kept in equilibrium by the large (0.1 mb) cross section for photodisintegration of deuterium

$$p+n \to d+\gamma+ 2.22 \text{ MeV}$$

(without this all n would decay and the Universe would be hydrogen).
The out-of-equilibrium temperature is not Q but $\simeq Q/40 \simeq 0.05$ MeV, because of the large photodisintegration σ.

4. After that all n, p transform into ^4He

5. nucleosynthesis "essentially" stops because there are no stable nuclei with $A = 5, 6$ or 8

$\boxed{\text{DARK MATTER}}$ (see slides)

- WMAP: dark matter in % of $\Omega_c = 9.9 \times 10^{-30} \frac{g}{cm^3}$

	$t = 380{,}000$ years	$t = t_0$
atoms	12	4.6
light	15	5×10^{-5}
neutrinos	10	$\lesssim 1\%$
dark matter	63	23
dark energy	negligible	72

- Relation $\Omega_{DM} \sim 1$ and $\sigma \sim 10^{-35}\,cm^2$ = WEAK !

$$\boxed{\left[N(T) = \left(\frac{MT}{2\pi}\right)^{3/2} e^{-\frac{M}{T}}\right]\left[G_F^2 M^2\right] \simeq H}$$

$$H = \frac{\dot{R}}{R} = \left[\frac{8\pi G}{3}\rho_r\right]^{1/2} = \left[\frac{4g\pi^3 G_N}{45}\right]^{1/2}(kT)^2$$

g for rad. era

solution $T \simeq \left[\frac{25}{M}\right]^{-1}$

$$N(0) = \left(\frac{T_0}{T}\right)^3 \frac{H}{\sigma v}$$

$\left(\frac{R}{R_0}\right)^3$

$\rho_{WIMP} = M N(0) =$

$$\boxed{\Omega_{WIMP} = \frac{\rho_{WIMP}}{\rho_{cr}} = \frac{10^{-25}}{\sigma v}\frac{cm^3}{s}}$$

at freeze-out $\frac{1}{2}Mv^2 = \frac{3}{2}kT$

- calculation of the recoil energy of a WIMP in a detector with nuclei of mass M ($\simeq mA$)

WIMP mass m and $v \simeq 10^{-3}c$ (galactic escape velocity)

Lorentz invariant c.m energy^2 in the collision

$$s = (p_\chi + p_A)^2$$

$$= p_\chi^2 + p_A^2 + 2E_\chi E_A - \cancel{2\vec{p}_\chi \vec{p}_A} \xrightarrow{} 0$$

evaluate in the lab system

$$= m^2 + M^2 + 2\,(\underbrace{m + \tfrac{1}{2}mv^2}_{E \text{ of the WIMP}})\,M$$

$$\sqrt{s} = \left[(m+M)^2 + 2ME\right]^{1/2} \quad \text{lab}$$

$$= \left(m + \frac{p^2}{2m}\right) + \left(M + \frac{p^2}{2M}\right) \quad \text{cm}$$

cm where the momenta are equal and opposite

also in CM, non-relativistic

p = c.m. momentum of each particle

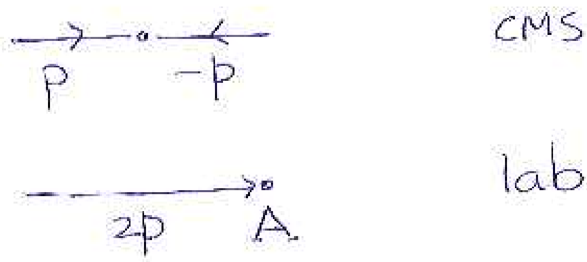
$$p^2 = \frac{2\mu^2 E}{M} = \mu^2 v^2 \text{ with } \quad \mu = \frac{mM}{m+M}$$

from $\sqrt{s}_{lab} = \sqrt{s}_{cm}$

- recoil of nucleus (this is what is measured)

$E < E_{max}$

$\hookrightarrow$ when $\cos\theta_{CM} = -1$



$$E_{MAX} = \frac{1}{2} M v_A^2 = \frac{p_A^2}{2M} = \frac{(2p)^2}{2M} = \frac{2p^2}{M}$$

$$p^2 = \mu^2 v^2$$

$$v = 10^{-3} c$$

$$\boxed{E_{MAX} \simeq A \text{ keV}}$$ or less