PHYSICS OF MASSIVE NEUTRINOS

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Plan of Lectures

I. Standard Neutrino Properties and Mass Terms (Beyond Standard)

II. Effects of ν Mass and Neutrino Oscillations (Vacuum)

III. Neutrino Oscillations in Matter

IV. The Emerging Picture and Some Lessons

Plan of Lecture I

Standard Neutrino Properties and Mass Terms (Beyond Standard)

Historical Introduction

The Standard Model of Massless Neutrinos

Mass-related Neutrino Properties:
Helicity versus Chirality, Majorana versus Dirac

Neutrino Mass Terms Beyond the SM:
Dirac, Majorana, the See-Saw Mechanism …

Discovery of ν's

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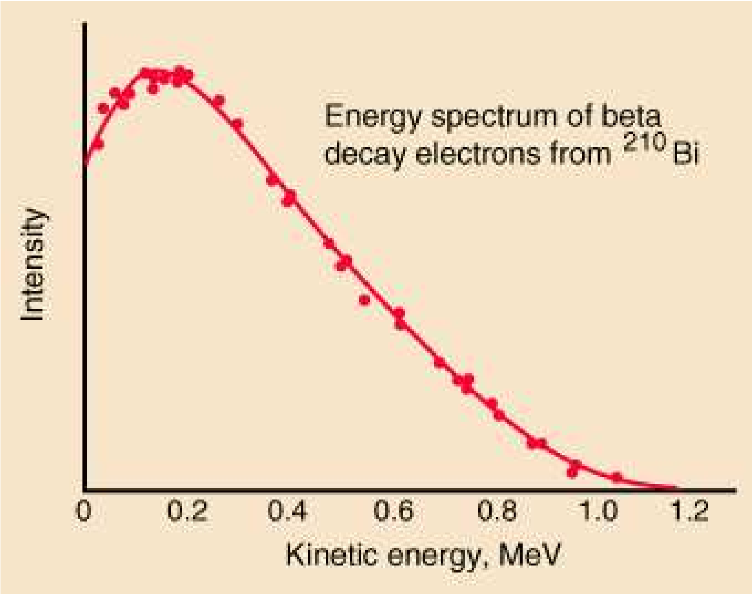

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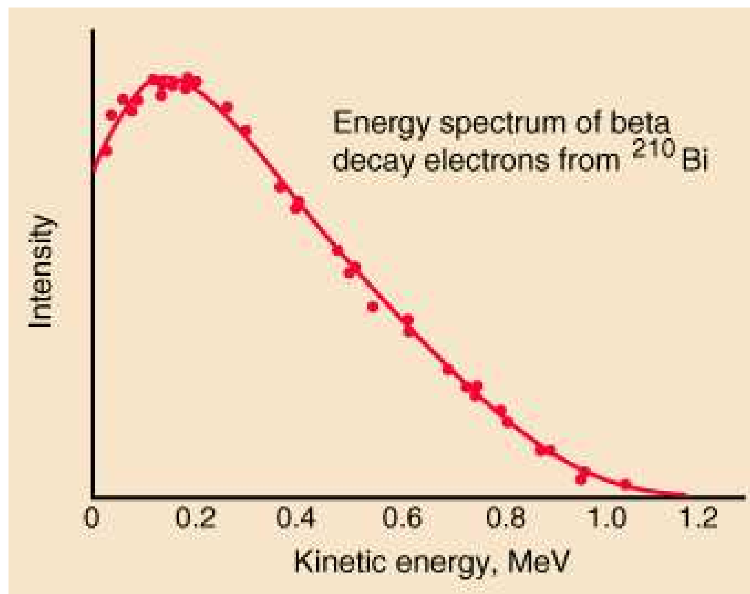

Do we throw away the energy conservation?

Discovery of ν's

• The idea of the neutrino came in 1930, when W. Pauli tried a desperate saving operation of "the energy conservation principle".

In his letter addressed to the "Liebe Radioaktive Damen und Herren" (Dear Radioactive Ladies and Gentlemen), the participants of a meeting in Tubingen. He put forward the hypothesis that a new particle exists as "constituent of nuclei", the "neutron" ν, able to explain the continuous spectrum of nuclear beta decay

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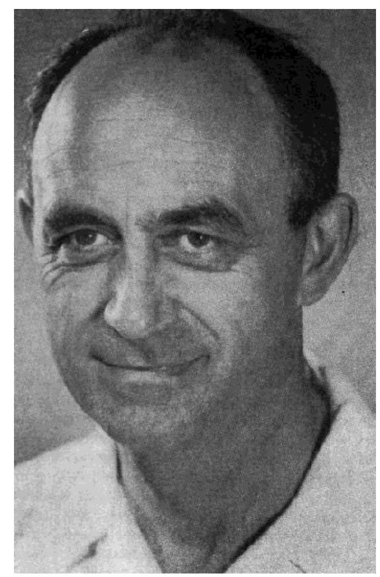

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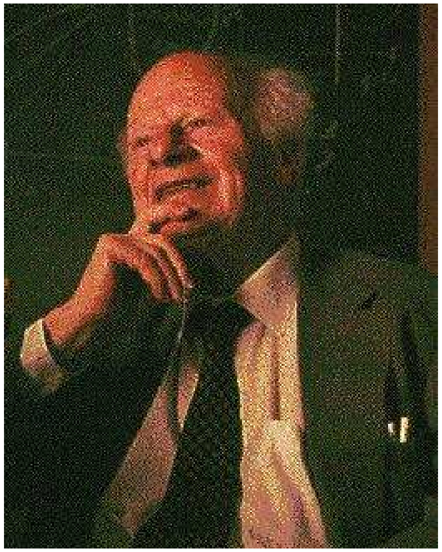

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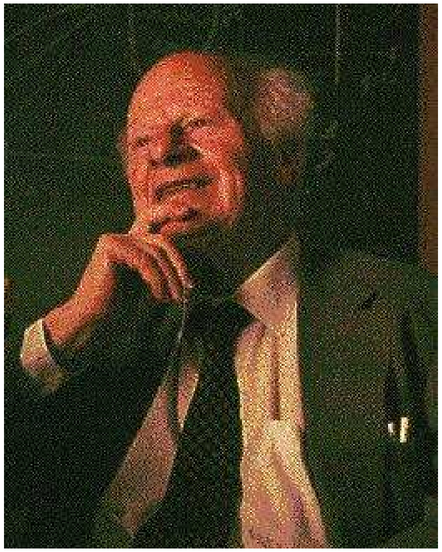

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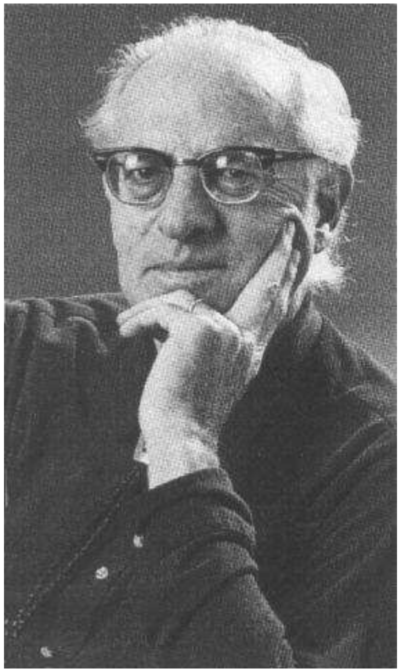

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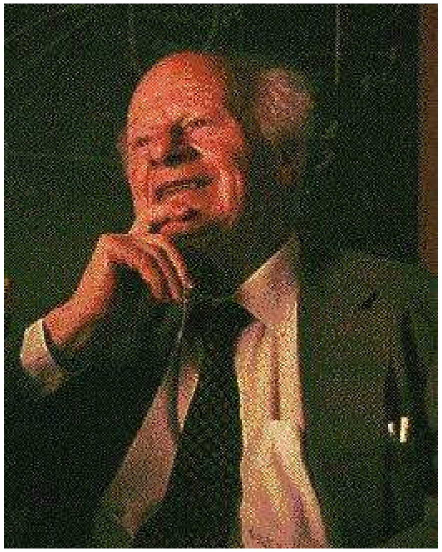

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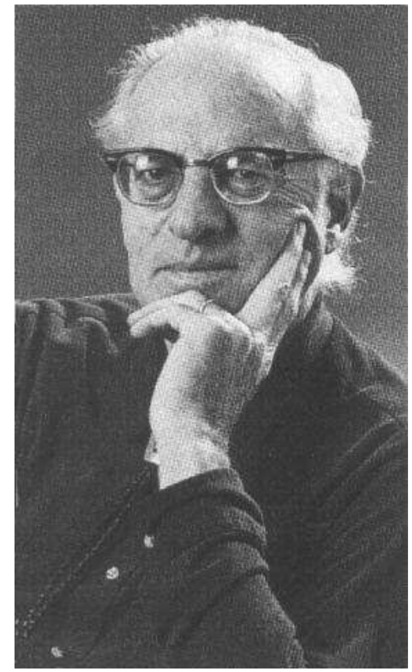

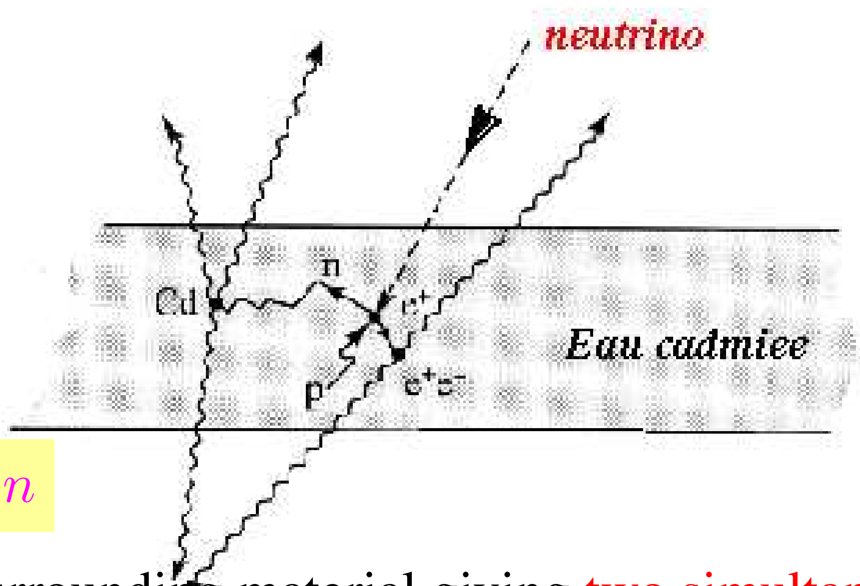

400 litters of water
and cadmium chloride.

$$\overline{\nu}_e + p \rightarrow e^+ + n$$

e^+ annihilates e^- of the surrounding material giving two simultaneous γ's.
neutron captured by a cadmium nucleus with emission of γ's some 15 msec after

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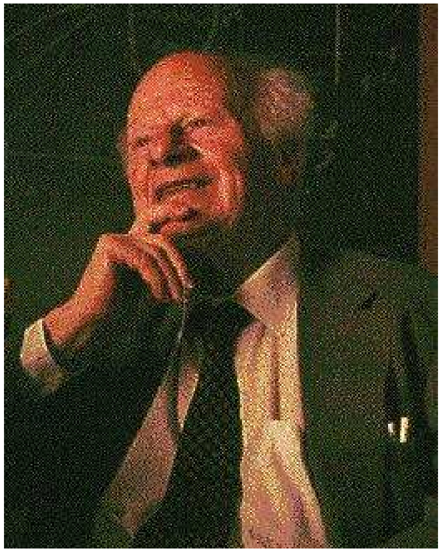

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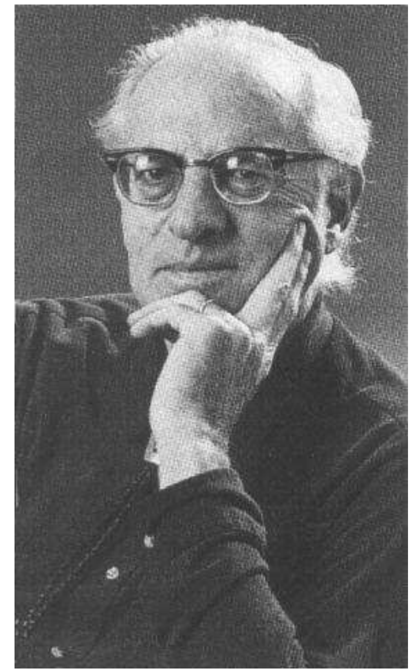

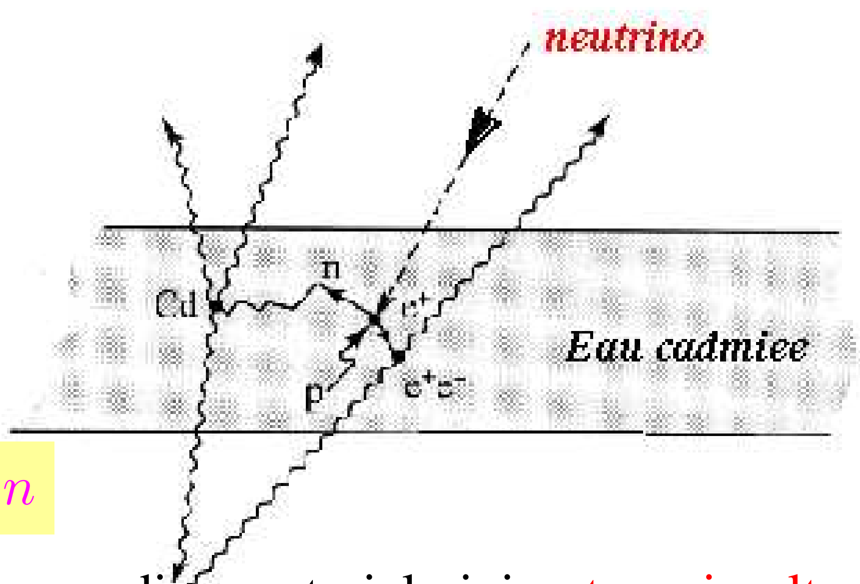

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The neutrino was there. Its tag was clearly visible

Neutrino Helicity

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- The neutrino helicity was measured in 1957 in a experiment by Goldhaber et al.

- Using the electron capture reaction

$$e^- + {}^{152}Eu \to \nu + {}^{152}Sm^* \to {}^{152}Sm + \gamma$$

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So momentum conservation $\Rightarrow \vec{p}(\nu) = -\vec{p}(\gamma) \Rightarrow$ ν helicity $=$ γ helicity

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- Goldhaber et al found γ had negative helicity $\Rightarrow$ ν has helicity -1

The Other Flavours

ν coming out of a nuclear reactor is $\overline{\nu}_e$ because it is emitted together with an e^-

Question: Is it different from the muon type neutrino ν_μ that could be associated to the muon? Or is this difference a theoretical arbitrary convention?

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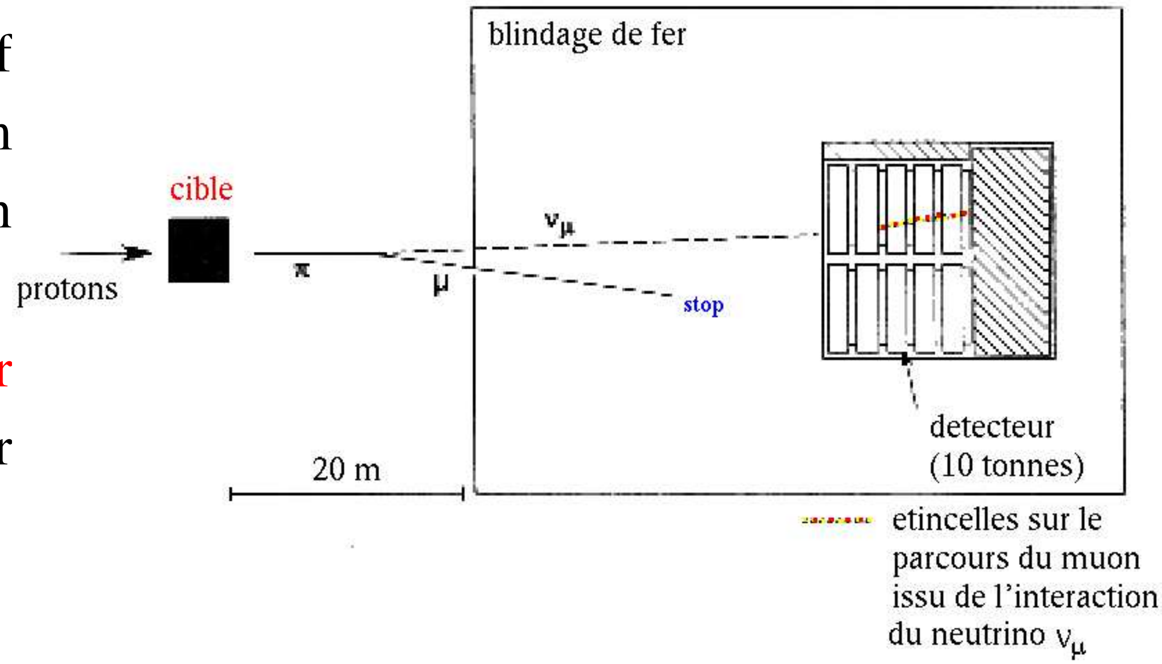
Schwartz, Lederman, Steinberger and Gaillard built a spark chamber (a 10 tons of neon gas) to detect ν_μ



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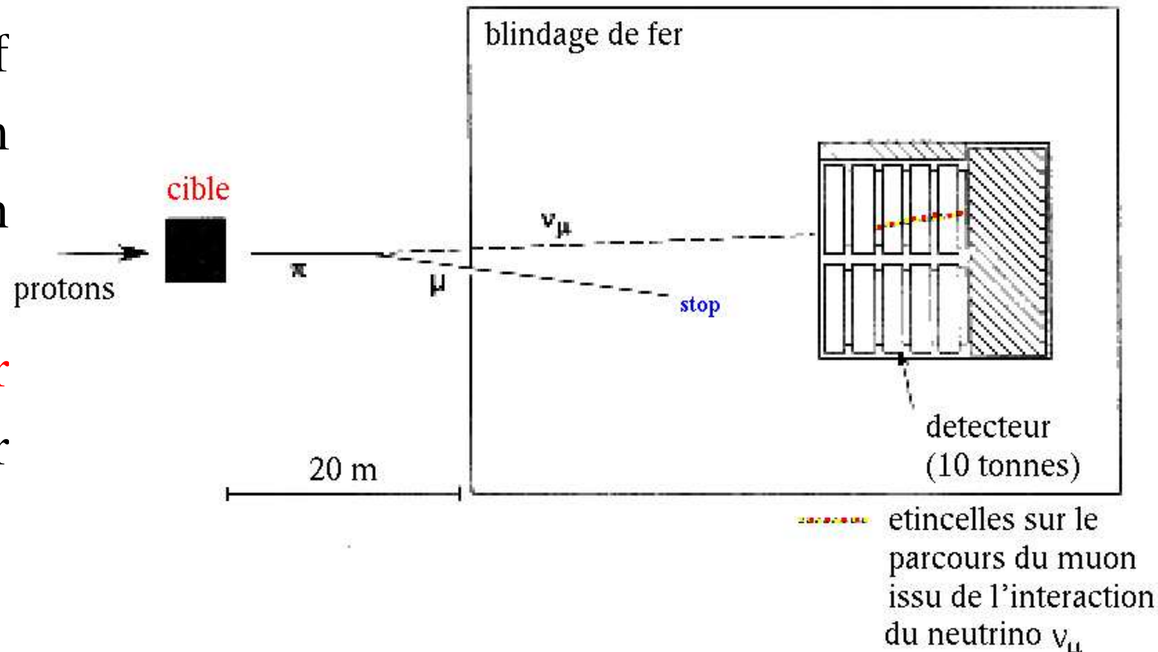

They observe 40 ν interactions: in 6 an e^- comes out and in 34 a μ^- comes out.

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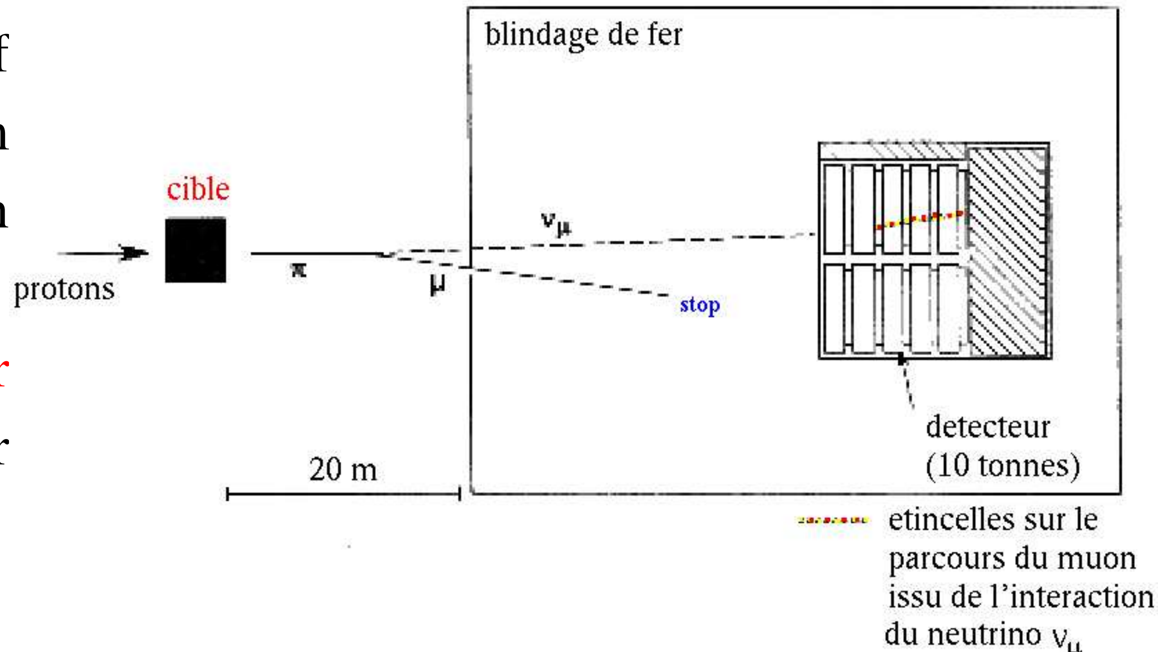

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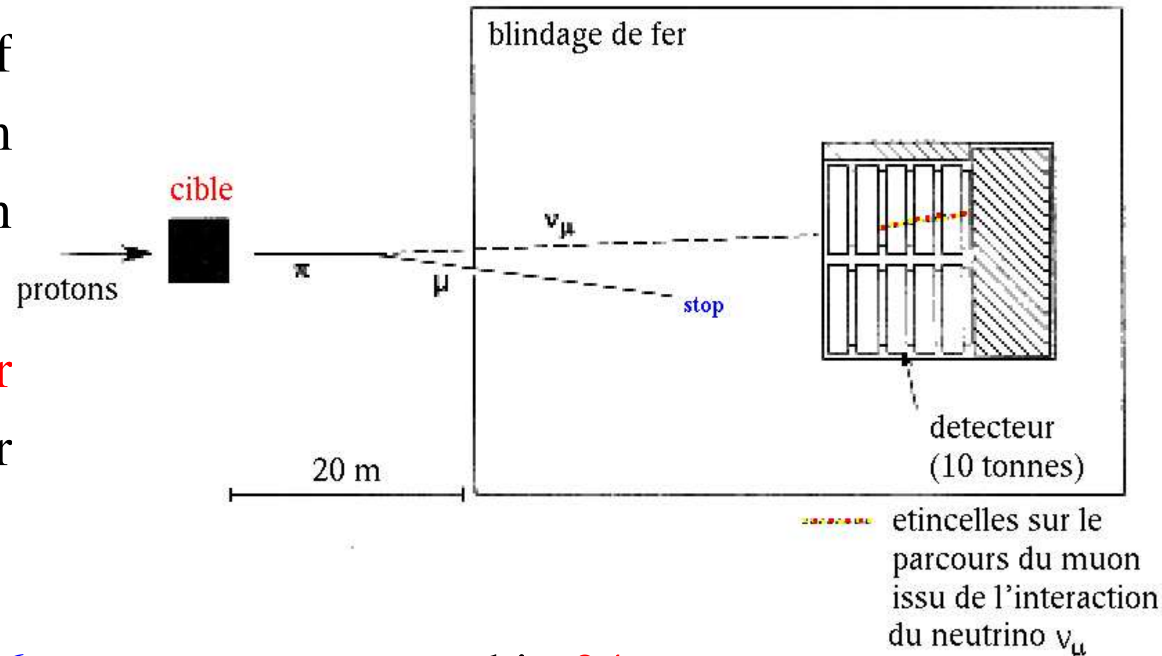

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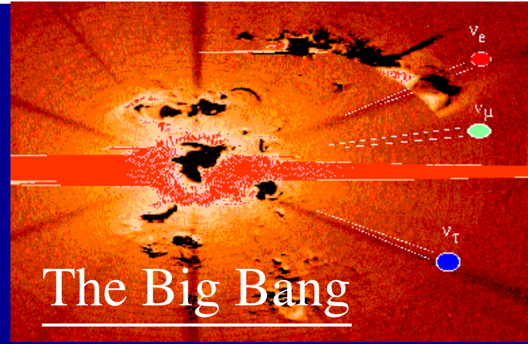
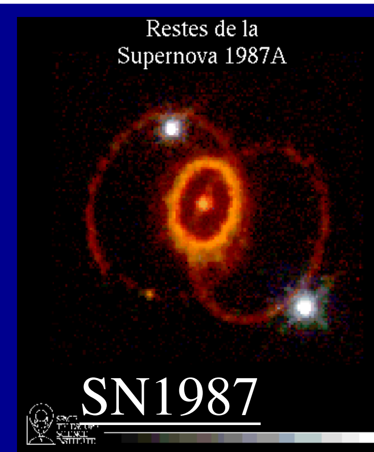
In 1977 Martin Perl discovers the particle tau $\equiv$ the third lepton family.

The ν_τ was observed by DONUT experiment at FNAL in 1998 (*officially* in Dec. 2000).

Sources of ν's



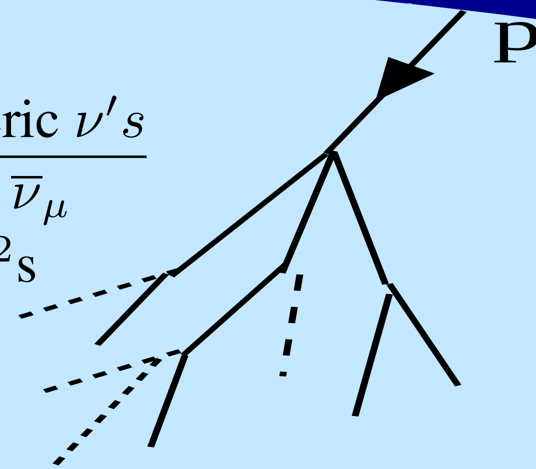
$\rho_\nu = 330/\text{cm}^3$
E_ν =0.0004 eV



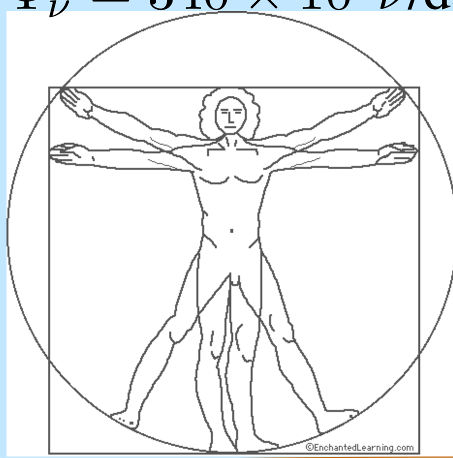
$E_\nu \sim$ MeV

The Sun
ν_e
$\Phi_\nu^{Earth} = 6 \times 10^{10} \nu/\text{cm}^2\text{s}$
$E_\nu \sim$ 0.1–20 MeV

Atmospheric $\nu's$
$\nu_e, \nu_\mu, \overline{\nu}_e, \overline{\nu}_\mu$
$\Phi_\nu \sim 1\nu/\text{cm}^2\text{s}$



Human Body
$\Phi_\nu = 340 \times 10^6 \nu$/day



Nuclear Reactors
$E_\nu \sim$ few MeV
$\overline{\nu}_e$

Fermilab
Discovering the Nature of Nature
CERN

Earth's radioactivity
$\Phi_\nu \sim 6 \times 10^6 \nu/\text{cm}^2\text{s}$

Accelerators
$E_\nu \simeq$ 0.3–30 GeV

KEK

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- 3 Generations of Fermions:

$(1,2,-\frac{1}{2})$ L_L	$(3,2,\frac{1}{6})$ Q^i_L	$(1,1,-1)$ E_R	$(3,1,\frac{2}{3})$ U^i_R	$(3,1,-\frac{1}{3})$ D^i_R
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} u^i \\ d^i \end{pmatrix}_L$	e_R	u^i_R	d^i_R
$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} c^i \\ s^i \end{pmatrix}_L$	μ_R	c^i_R	s^i_R
$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$\begin{pmatrix} t^i \\ b^i \end{pmatrix}_L$	τ_R	t^i_R	b^i_R

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- Spin-0 particle ϕ: $(1,2,\frac{1}{2})$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \overset{SSB}{\longrightarrow} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

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$$Q_{EM} = T_{L3} + Y$$

- ν's are $T_{L3} = \frac{1}{2}$ components lepton doublet L_L
- ν's have no strong or EM interactions
- No ν_R (they are singlets of gauge group)

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SM Fermion Lagrangian

$$
\begin{aligned}
\mathcal{L} \; = \; & \sum_{k=1}^{3}\sum_{i,j=1}^{3} \overline{Q^i_L}\gamma^\mu \left(i\partial_\mu - g_s \tfrac{\lambda_{a,ij}}{2} G^a_\mu - g \tfrac{\tau_a}{2}\delta_{ij} W^a_\mu - g' \tfrac{Y}{2}\delta_{ij} B_\mu\right) Q^j_{L,k} \\
& \sum_{k=1}^{3}\sum_{i,j=1}^{3} + \overline{U^i_{R,k}}\gamma^\mu \left(i\partial_\mu - g_s \tfrac{\lambda_{a,ij}}{2} G^a_\mu - g' \tfrac{Y}{2}\delta_{ij} B_\mu\right) U^j_{R,k} \\
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& \sum_{k=1}^{3} + \overline{L_{L,k}}\gamma^\mu \left(i\partial_\mu - g \tfrac{\tau_i}{2} W^i_\mu - g' \tfrac{Y}{2} B_\mu\right) L_{L,k} + \overline{E_{R,k}}\gamma^\mu \left(i\partial_\mu - g' \tfrac{Y}{2} B_\mu\right) E_{R,k} \\
& - \sum_{k,k'=1}^{3} \left(\lambda^u_{kk'} \overline{Q}_{L,k}(i\tau_2)\phi U_{R,k'} + \lambda^d_{kk'} \overline{Q}_{L,k}\phi D_{R,k'} + \lambda^l_{kk'} \overline{L}_{L,k}\phi E_{R,k'} + h.c.\right)
\end{aligned}
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& - \sum_{k,k'=1}^{3} \left(\lambda^u_{kk'} \overline{Q}_{L,k} (i\tau_2)\phi U_{R,k'} + \lambda^d_{kk'} \overline{Q}_{L,k}\phi D_{R,k'} + \lambda^l_{kk'} \overline{L}_{L,k}\phi E_{R,k'} + h.c. \right)
\end{aligned}
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- Invariant under global rotations

$$
q_i \to e^{i\alpha_B/3} q_i \qquad l_i \to e^{i\alpha_{L_i}/3} l_i \qquad \nu_i \to e^{i\alpha_{L_i}/3} \nu_i
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SM Fermion Lagrangian

$$
\begin{aligned}
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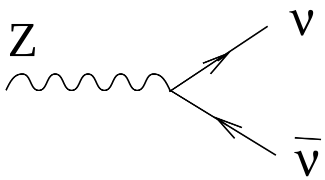
$\Rightarrow$ *Accidental (≡ not imposed)* global symmetry: $B \times L_e \times L_\mu \times L_\tau$

$\Rightarrow$ Each lepton flavour, L_i, is conserved

$\Rightarrow$ Total lepton number $L = L_e + L_\mu + L_\tau$ is conserved

Number of Neutrinos

• The counting of light active left-handed neutrinos is based on the family structure of the SM assuming a universal diagonal NC coupling:



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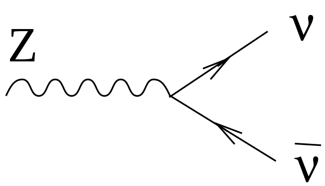

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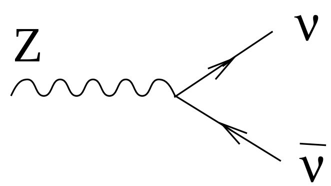

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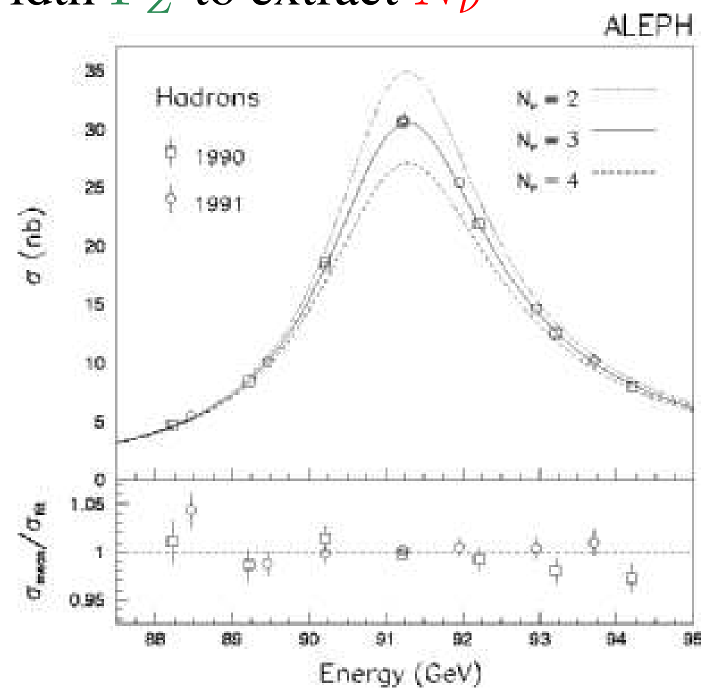

Leads $N_\nu = 2.9840 \pm 0.0082$

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$$m_f \overline{f_L} f_R + h.c. \quad \text{(this is not } SU(2)_L \text{ gauge invariant)}$$

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In SM Neutrinos are *Strictly* Massless

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• However what Goldhaber measured was the helicity not the chirality of ν

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Only for massless fermions Helicity and chirality states are the same.

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- In the SM neutral bosons can be of two type:
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$$\nu^C = \mathcal{C}\,\nu\,\mathcal{C}^{-1} = \eta_C^\star \sum_{s,\vec{p}} \left[b_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + a_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right] = -\eta_C^* \, C\, \overline{\nu}^T$$

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$\Rightarrow$ These two fields can rewritten in terms of 4 chiral fields

$\nu_L\ ,\ \nu_R\ ,\ (\nu_L)^C\ ,\ (\nu_R)^C$ with $\nu = \nu_L + \nu_R$ and $\nu^C = (\nu_L)^C + (\nu_R)^C$

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Only involves two chiral fields $P_L\,\nu = \nu_L$ and $\overline{\nu}\,P_R = \eta_C (\nu_L)^{C^T} C^\dagger$

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$$\Rightarrow \eta_C^\star \sum_{s,\vec{p}} \left[b_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + a_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right] = \sum_{s,\vec{p}} \left[a_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + b_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right]$$

$\Rightarrow$ So we can rewrite the field $\nu_M = \sum_{s,\vec{p}} \left[a_s(\vec{p}) u_s(\vec{p}) e^{-ipx} + \eta_C^\star a_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ipx} \right]$

which contains only one set of creation–annihilation operators

$\Rightarrow$ A Majorana particle can be described with only 2 independent chiral fields:

ν_L and $(\nu_L)^C$ which verify $\nu_L = (\nu_R)^C$ $(\nu_L)^C = \nu_R$

- In the SM the interaction term for neutrinos

$$\mathcal{L}_{int} = \frac{i\,g}{\sqrt{2}}[(\bar{l}_\alpha \gamma_\mu P_L \nu_\alpha) W_\mu^- + (\bar{\nu}_\alpha \gamma_\mu P_L l_\alpha) W_\mu^+] + \frac{i\,g}{\sqrt{2}\cos\theta_W}(\bar{\nu}_\alpha \gamma_\mu P_L \nu_\alpha) Z_\mu$$

Only involves two chiral fields $P_L\,\nu = \nu_L$ and $\overline{\nu}\,P_R = \eta_C (\nu_L)^{C^T} C^\dagger$

$\Rightarrow$ Weak interaction cannot distinguish if neutrinos are Dirac or Majorana

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The difference arises from *the mass term*

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- A fermion mass can be seen as at a Left-Right transition

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How can we generate a mass for the neutrino?

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OPTION 1:

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$\Rightarrow$ Total Lepton number is conserved *by construction* (not *accidentally*):

$$U(1)_L\,\nu = e^{i\alpha}\,\nu \quad \text{and} \quad U(1)_L\,\overline{\nu} = e^{-i\alpha}\,\overline{\nu}$$

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⇒ But $SU(2)_L$ gauge invariance is broken!!!

General $SU(2)_L$ invariant ν Mass Terms

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- In general if $M_N \neq 0 \Rightarrow$ 3+m Majorana neutrino states

$$\nu^M = V^{\nu\,\dagger}\nu_L + (V^{\nu\,\dagger}\nu_L)^c \quad \text{(verify } \nu^{M\,c}_i = \nu^M_i \text{)}$$

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- A particular realization of OPTION 3: Add m ν_{R_i} so

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  - Arises in many extensions of the SM: SO(10) GUTS, Left-right...

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  - Accidental global symmetry: $B \times L_e \times L_\mu \times L_\tau \leftrightarrow m_\nu \equiv 0$
  - neutrinos are left-handed ($\equiv$ helicity -1): $m_\nu = 0 \Rightarrow$ chirality $\equiv$ helicity
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  Answer:

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- In the SM:
  - Accidental global symmetry: $B \times L_e \times L_\mu \times L_\tau \leftrightarrow m_\nu \equiv 0$
  - neutrinos are left-handed ($\equiv$ helicity -1): $m_\nu = 0 \Rightarrow$ chirality $\equiv$ helicity
  - No distinction between Majorana or Dirac Neutrinos
- If $m_\nu \neq 0 \rightarrow$ Need to extend SM

  $\rightarrow$ different ways of adding m_ν to the SM
  - *breaking* total lepton number ($L = L_e + L_\mu + L_\tau$) $\rightarrow$ Majorana ν: $\nu = \nu^C$
  - *conserving* total lepton number $\rightarrow$ Dirac ν: $\nu \neq \nu^C$
- Question: How to search for m_ν?

  Answer: Tomorrow....