- the cosmic fluid

up to now we have described a Universe of non-relativistic matter (Newtonian limit)

thermodynamic of the perfect cosmic fluid:

$$dE = -P\,dV \qquad V = \frac{4\pi}{3}R^3$$

$$E = \underbrace{\rho V}_{M} c^2$$

$$d(\rho c^2 R^3) = -P\,dR^3$$

$$\dot{\rho} = -3\left(\frac{\dot{R}}{Rc^2}\right)(\rho c^2 + P)$$

combine with Friedman II

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G}{3}\rho$$

↓ differentiate

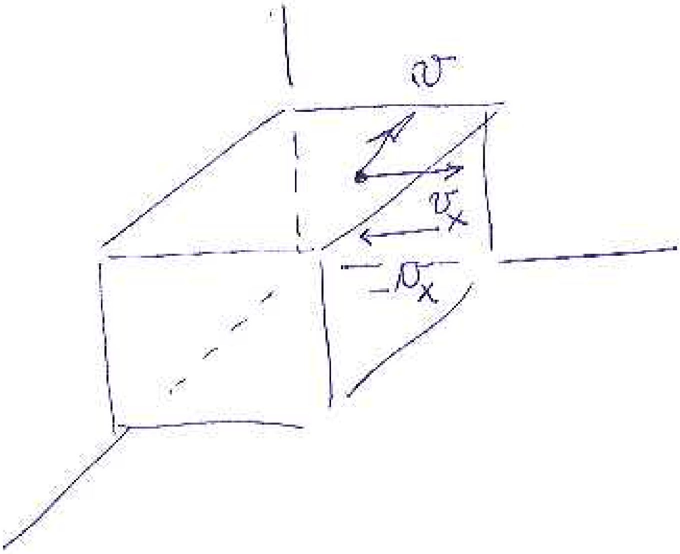
$$2\dot{R}\ddot{R} = \frac{8\pi G}{3}\left(\dot{\rho}R^2 + 2\rho R\dot{R}\right)$$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right)$$

particles exert a pressure as well. Friedman I with

$$\rho \to \rho + \frac{3P}{c^2}$$

- pressure of N relativistic particles in a box or equation of state of the Universe




$$P = \frac{F}{L^2}$$

$$= \frac{N}{L^2} \frac{\Delta p}{\Delta t}$$

part. moves 1/3 of time in x-dir

$$= \frac{N}{L^2} \frac{m\,(2v_x)}{\frac{2L}{v_x}} \; \frac{1}{3}$$

$$\boxed{P = \underbrace{\left(\frac{N}{L^3}\right)}_{n} m v^2 \, \frac{1}{3}}$$

energy density

$$\rho = \frac{1}{2} n m v^2$$

$$P = \frac{2}{3}\rho = \frac{2}{3}\rho c^2 \left(\frac{v^2}{c^2}\right)$$

this is non-relativistic. Relativistic

$$\rho c^2 = n\, m c^2 = n \langle pc \rangle$$

radiation pressure

$$P_{rel} = \frac{1}{3}\rho c^2$$

- pressure of the vacuum

- conclusion: include all forms of energy density in the RHS of Friedman Eq.

- radiation in the Universe ($\gamma, \nu$ dominated early !)

$$\rho_r c^2 = g a T^4 \qquad a = 7.57 \times 10^{-16}\ J m^{-3} K^{-4}$$

$g$ = number of spin states (e.g 2 for photon)

Friedman II (radiation)

$$\boxed{\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3c^2}\, g a T^4}$$

$$\rho_r \sim \frac{1}{T^4} \sim \frac{1}{R^4} \sim \frac{1}{R^3}\,\frac{1}{R}$$

$E \sim \frac{1}{\text{wavelength}}$

wavelength expands with R

$$\frac{T}{T_0} = \frac{R_0}{R}$$

$$R = R_0 T_0 \frac{1}{T}$$

$$dR = -R_0 T_0 \frac{dT}{T^2} = -RT\frac{dT}{T^2}$$

Friedman II $$\frac{dR}{dt} = R\left[\frac{8\pi G}{3c^2}\, g a T^4\right]^{1/2}$$

$$dR = -RT\frac{dT}{T^2} = R\left[\frac{8\pi G}{3c^2}\, g a\right]^{1/2} T^2\, dt$$

$$-\int \frac{dT}{T^3} = \left[\quad\right]^{1/2} t = \frac{1}{2T^2}$$

$$\boxed{T = \left[\frac{3c^2}{32\pi G a}\right]^{\frac{1}{4}} t^{-\frac{1}{2}}}$$

- practical forms (t in seconds)

$$T_{Kelvin} = \frac{1.52\times10^{10}}{g^{1/4}\, t^{1/2}}$$

$$t = \frac{12}{g^{1/2}\, E^2_{MeV}}$$

$$\frac{R}{R_0} = \frac{T_0}{T} = 1.81\times10^{-10}\, g^{1/4}\, t^{1/2}$$

$$g = n_{\text{bosons}} + \frac{7}{8}\, n_{\text{fermions}} = \begin{cases} 2 & \text{for } \gamma \\ \frac{11}{2} & \text{for } \gamma, e^{\pm} \\ \frac{43}{4} & \text{for } \gamma, e^{\pm}, 3\nu \\ \vdots & \end{cases}$$

- hot big bang

decoupling $\quad$ reaction rate $\simeq$ expansion rate

$$\sigma = \frac{\Gamma}{\text{flux}} = \frac{\Gamma}{n v}$$

$$\boxed{\Gamma = \underbrace{\langle n \sigma v \rangle}_{\text{thermal distribution}} \simeq H = \frac{\dot{R}}{R}}$$

$$\underbrace{e \quad p}_{H} \quad \gamma$$

photons fall out of equilibrium when they cannot ionize H anymore ( $E_{MeV} < 13.6 \cdot 10^{-6}$ ). Photons expand with the Universe after that without interacting creating the CMBR. The expansion is matter dominated after decoupling.

- comment on 13.6 eV : it is an overestimate

photons of lower energy can still ionize excited H
photons in the tail of the thermal distribution can ionize H even at a later "time" or lower average temperature

$$\boxed{13.6\ \text{eV} \rightsquigarrow 0.3\ \text{eV}}$$

◎ games with numbers

critical $\rho_c \cong \cancel{2 \times 10^{-29}}\ 9.9 \times 10^{-30}\ \frac{g}{cm^3}$

4%

$$\rho_{m_0} = \Omega_0 \rho_c c^2 \simeq 10^{-10}\ \frac{J}{m^3} \quad \text{for } \Omega_0 \simeq 0.04$$

$$\rho_{r0} = 4.3 \times 10^{-14}\ \frac{J}{m^3} \qquad \Omega_r \simeq 5 \times 10^{-5}$$

at matter-radiation transition $\rho_m \simeq \rho_r$ this is roughly the time of decoupling

$$\rho_{m0} \left(\frac{R_0}{R_d}\right)^3 \simeq \rho_{r0} \left(\frac{R_0}{R_d}\right)^4$$

$$\frac{R_0}{R_d} = 2000$$

decoupling time

$$t_d = \frac{12}{g^{1/2} E^2_{MeV}} \cong 10^{13}\,s \simeq 300'000 \text{ years}$$

$$\left[ = \frac{1.2 \times 10^{13}\,s}{g^{1/2} E^2_{eV}} \right]$$

$<2$ $\qquad$ $0.3 - 13.6$

$$T_k = \frac{1{,}52 \times 10^{10}}{g^{1/4}\, t^{1/2}} \cong \frac{1{,}52 \times 10^{10}}{t^{1/2}} \cong 5000\,°K$$

$$T_{k0} = 5000\,°K \cdot \underbrace{\frac{R_d}{R_0}}_{\frac{1}{2000}} = 2{,}5\,°K \quad \checkmark$$

number of baryons in today's Universe $\frac{\rho_{m0}}{m_{proton}} = 10^{-7}\,cm^{-3}$

$\frac{n_b}{n_\gamma} \cong 5 \cdot 10^{-10}$ (nucleosynthesis)

$n_\gamma = 2 \cdot 10^{9} \cdot 10^{-7}\,cm^{3} \simeq 200\,cm^{-3}$ (actually 410)